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Subject: Fall 2006 Written Candidacy Exam

From: *S. L. Garrett, Chairman of Candidacy Committee*

To: Students taking Fall 2006 Acoustics Program Candidacy Exam

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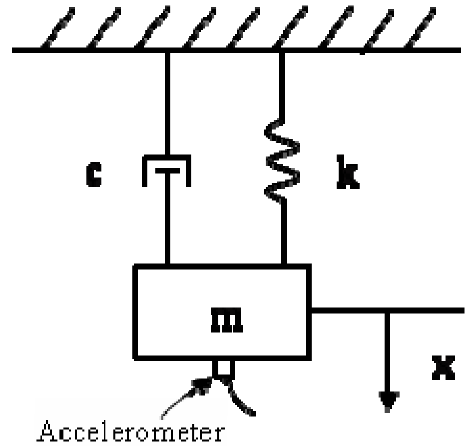
You have four (4) hours to complete the questions given below. This is a CLOSED BOOK exam; however, you may use up to five (5) pages of hand-written notes (a page is defined as one side of an 8.5-inch x 11-inch piece of paper). Please start each question on a clean sheet of paper, being sure to put your name at the top of each page and the number of the problem being worked out. Be neat and precise with your penmanship. Do not re-derive equations that are known, unless the question asks you to do so.

1. **Mechanical Loss Factor:** A simple one-degree-of-freedom mass-spring-damper system is shown in the figure. The governing differential equation for this system is given by:

$$m\ddot{x} + c\dot{x} + kx = 0,$$



and the solution to this equation can be given by:

$$x(t) = Ae^{-\zeta\omega_n t}\cos(\omega_d t + \varphi),$$

where,

ω_n is the undamped natural frequency, ω_d is the damped natural frequency, and ζ is the critical damping ratio. Also,

$$\zeta = \frac{c}{2\sqrt{mk}}, \quad \omega_n = \sqrt{\frac{k}{m}}, \quad \omega_d = \omega_n\sqrt{1-\zeta^2}.$$

Assume the system has damping lower than critical damping, i.e. $\zeta < 1$ and the initial conditions are x(0)=1, and u(0)=0 (where u is velocity).

a.) For $t \geq 0$ sketch what the time trace from the accelerometer would look like on an oscilloscope. Show the free-response decay envelope. What is the equation for the free response decay envelope?

b.) Mentally, take the absolute value of the time trace from part a.) and then take the natural logarithm. Sketch what the result would look like. (Hint: Think about the free response decay envelope.)

c.) Using the result in part b.), and any of the formulas above that you need, derive the equation for calculating the critical damping ratio based upon the accelerometer measurement.

2. **Circular Membrane:** A circular membrane fixed at its rim has a radius of 1 meter and a surface density of 0.1 kg/m^2. The membrane is struck in the middle. Ignore air loading effects and determine:

a.) the uniform tension for which the lowest of the excited frequencies is 256 Hz

b.) using the tension derived in part (a), the frequency of the first overtone.

The roots of $J_0(x) = 0$ are 2.405, 5.520, 8.654, ….., and the roots of $J_1(x) = 0$ are 3.83, 7.02, 10.17,…..

3. **Guitar String:** The density of steel is 7.7 gm/cm^3 and its yield strength is 200,000 $pounds/in^2$. What is the highest value of the fundamental frequency of a steel guitar string, rigidly fixed at both ends, which is 80 cm in length?

4. **Half-wave Rectified Waveform:**

A sinusoidal voltage,

$$V = E\sin(\omega t)$$

is passed through a half-wave rectifier which clips the negative portion of the wave. Find the Fourier series of the resulting periodic function.

5. **Complex Numbers:** Evaluate the following, where i is the unit imaginary number:

a.) $\ln(3+4i)$

b.) $\sqrt[3]{1}$

c.) $\left|\dfrac{a-ib}{b-ia}\right|$

6. **Snell's Law:** A plane acoustic wave is incident on the ocean floor at an angle of 55^o. The ocean floor is quartz sand having a density of 2070 kg/m^3 and a speed of sound of 1730 m/s. The density and sound speed of seawater are 1026 kg/m^3 and 1500 m/s, respectively.

a.) Calculate the angle of transmission.

b.) If the incident wave has a peak pressure amplitude of 100 Pa, what is the SPL (re 1 μ Pa) of the reflected wave?

c.) What is the SPL (re 1 μ Pa) of the transmitted wave?

7. **Bubble Resonance:** A spherical bubble of equilibrium radius, *a*, suspended in an incompressible liquid medium can be considered a simple oscillator. At equilibrium, the hydrostatic pressure of the surrounding medium is balanced by the gas pressure inside the bubble. For a incident acoustic plane wave of sound pressure, $p_i(t)$, with a wavelength much larger than the bubble radius, the force at the bubble surface is:

$$f = -p_i A = -s\xi \, .$$

where A is the surface area of the bubble, ξ represents the radial displacement and s is the equivalent stiffness of the system. When the bubble is subjected to a pressure change, its volume will change according to the ideal gas law, which for the adiabatic case yields the relation

$$pV^{\gamma} = \text{constant}$$

where p is the external pressure, V is the volume and γ is the ratio of specific heats.

a.) Determine an expression for the equivalent stiffness for this system. (Hint: the pressure change, dp will be equal to p_i, the pressure p is approximately equal to the ambient pressure p_a and the volume change, dV, equals $4\pi a^2\xi$).

b.) Determine an expression for the radiation mass for this system. (Hint: when the bubble is oscillating, it acts like a small pulsating sphere, radiating sound uniformly in all directions).

c.) Finally, using the results of a) and b) above determine the resonance frequency for a bubble of radius a.

8. **Bubble Radiation:** A simple bubble in a liquid is in dynamic motion and radiates according to:

$$p(r) = \frac{\partial}{\partial t}\int_S \frac{\rho v_n}{4\pi r} dS$$

where v_n is the normal surface velocity of the bubble, r is the far-field radial observation position, t is time, and S is the surface area of the bubble. Do an order-of-magnitude analysis of this equation for a single bubble of radius a. This question is asking for an *approximate* expression for *the magnitude* of p at location r that depends upon the bubble radius a and the surface velocity, v_n.