

L^AT_EX and Writing Mathematics

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May 3, 2005

Acknowledgements

This document is dedicated to Dr. Jim DeFranza, who has been my advisor during my four years at St. Lawrence. He has not only been my professor, but also a mentor and a friend. Without his guidance, I would never have acquired any of the knowledge, inspiration, or vision to write this document. I want to thank him for the hard work, time, and effort that he has put into my honors project.

Preface

The course in technical writing was my first real experience in communicating mathematics through clear and concise writing. This has had a significant positive effect on not only my mathematical writing, but also my ability to give mathematical arguments. The goal of this project is to create a manual that will share these experiences with other students. The targeted audience will be perspective mathematics majors who are either advanced first year college students or second year college students who are enrolled in the Bridge to Higher Mathematics course. The document will be written in the context of how to write proofs, to go along with the topics in the Bridge course (MATH 280). It will include how to use the program \LaTeX for typing math documents. It will also include examples and tutorials, such as what one should and should not do when writing mathematics. The use of Maple will be included as well, for creating graphics and doing computations. This document can also be used as a model for a $\frac{1}{2}$ credit course to be taken simultaneously with the Bridge course in the future.

Chapter 1 Introduces the practical implications of using writing in the field of mathematics. It outlines the way that writing can and should be used in mathematics and then goes further to give examples of what is good and what is bad.

Chapter 2 Describes how to obtain the necessary programs, explains the basic functions and most common commands of \LaTeX that are necessary in all documents. It walks a student through the process of creating and compiling a document for the first time.

Chapter 3 Explores some of the more complex features of \LaTeX that are used when creating larger documents that require a more elaborate format, such as an honors thesis like this.

Chapter 4 Delves into the material from the course MATH 280: Bridge to Higher Mathematics where the most common proof techniques are outlined and examples are given for further clarification.

Chapter 5 Explains the use of the programs Maple and Adobe Illustrator to create and modify graphics. It then goes further to show how such graphics are inserted into a \LaTeX document.

Chapter 6 Discusses the process of creating this document along with the reasons it is important. It includes the intermediate steps along the way as well as the successes and failures of the project - what was easy and what was more difficult to accomplish.

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Chapter 1

Introduction to Writing Mathematics

1.1 Introduction

This manual has been written to explain the use of the program called \LaTeX , and describe how mathematical arguments are clearly written. The audience to which this document is directed is students currently enrolled in Mathematics 280: Bridge to Higher Mathematics. The reason for choosing this particular audience and the content of this document will be described in this chapter.

1.2 Rationale

Why is it necessary to write a manual about how to use the program \LaTeX to write mathematics properly? There is no quick and easy answer to this question, but it is an extremely important topic. First and foremost, at the college level, mathematics is not merely a collage of numbers and symbols. Students who are interested in pursuing a math major should be able, by the time they get to this level of math, to articulate their thoughts, explain their work, and write clear and comprehensive mathematical arguments.

Communication is important in every field, especially math and the sciences. Math is not only about memorizing formulas, understanding the concepts, and being able to solve the problems. The way to prove that you have a true understanding of math is to show an understanding of the problem at hand, to explain your thoughts, and to describe the processes used to arrive at the final solution. Although most students may not immediately realize it, writing complete explanations of math adds to the student's learning and understanding of the assignment.

1.3 Practical Implications

Writing is always done within some context and it is important to carefully consider the audience towards whom the document is directed. One can easily see that the language and words used to address third grade students would be drastically different from those used in assignments for college professors. A good thing to keep in mind when trying to explain arguments is the amount of detail that is necessary to clearly portray what you are trying to say. In order to write effectively you must have something to say and have a clear enough understanding to have the confidence to say it and convince the reader it is worth saying.

There should be enough information so that the document will make sense, to the author or anyone else reading it, in a few years, but the best writing is concise. There is a happy medium that will eventually be easier to find with time, as one's skills improve. The only way to build such skills is practice. The more you write, the better your writing will be.

Keep in mind that math documents follow the same rules as assignments in other subjects, such as English papers or scientific lab reports. Whenever formally writing up a solution or problem set, the basic guidelines should be followed. Every sentence should start with a capital letter and end with a period. Complete sentences are necessary and correct punctuation accompanies them.

Also, writing is a process. The first draft of any piece of writing is never the best version. Any document should be printed out and read over a few times, or at least once, reading to check for content, spelling, grammar, and punctuation. Read it out loud to yourself and listen. Does it sound right? Make sure that every one of your sentences begins with a capital letter, uses appropriate punctuation inside of it, and ends with a period. Your document should not include any mathematical equations or statements that stand alone. Calculations without any context or explanation omit ideas. Each and every part of the document should be included in a sentence somewhere. The ability to think clearly and the ability to write clearly are not separate activities.

1.4 A Writing Checklist

Every time you go to write any sort of document for a math course, whether it be something as simple as a homework assignment or as important as a take home exam, you should consider the following checklist to make sure that you have done all the necessary steps.

- Have you clearly stated the problem to solve?
- Is there an Introduction? Is there a paragraph that explains the approach that will be taken to solve the problem?
- Is the answer explicitly stated in one or more complete sentences?

- Have you clearly stated all the assumptions?
- Have you defined all the variables used in the solution?
- Have all figures and tables been clearly labeled?
- Have all outside sources of information used been referenced properly?
- Have you explained how each formula used is derived or where it can be found?
- Have you checked the spelling, grammar and punctuation?
- Have you checked that all mathematical symbols are used correctly?
- Have you checked that the mathematics is correct?
- Have you solved the question that was asked?
- Have you reread your solution and can you follow every line of the logic in your argument?

1.5 Final Advice on Writing Math

There are many common errors found in student writing, so just to make you aware of the mistakes NOT to make, we have included for you a list of tips when writing. These are all things to keep in the back of your mind when you write up a homework assignment or any sort of document for a math class.

1. The standard rules still apply: make sure your spelling, grammar, and punctuation are all correct. Don't forget that a sentence begins with a capital letter and ends with a period. Mathematical expressions can be included as part of the sentence, they shouldn't be left in the middle of the page without punctuation.

Bad: the solution to the equation $2x + 2 = 0$ is $x = -1$
 Good: The solution to the equation $2x + 2 = 0$ is $x = -1$.

2. It may seem trivial, but use complete sentences.
3. Mathematical variables should be italicized, or enclosed in dollar signs if you are using L^AT_EX.

Bad: If $x = 2$, then $x^2 = 4$.
 Good: If $x = 2$, then $x^2 = 4$.

4. Do not merely use commas to separate numerous formulas, use words to help clarify what the reader is seeing.

Bad: If $x = 5$, $2x + 3 = 13$.

Good: If $x = 5$, then $2x + 3 = 13$.

5. Start all sentences with words, not symbols.

Bad: $f(x) = (x - 2)^2 + 1$ has a minimum point at $(2, 1)$.

Good: The function $f(x) = (x - 2)^2 + 1$ has a minimum point at $(2, 1)$.

6. Don't be ambiguous, clearly define or explain what all symbols represent.

Bad: The surface area of a right circular cylinder is $2\pi rh + 2\pi r^2$.

Good: A right circular cylinder, with height h and radius of its top r , has surface area $2\pi rh + 2\pi r^2$.

7. State any and all assumptions that are being made.

8. Use a new symbol or letter for each variable you have. Once a variable name has been assigned do not re-use it for a different meaning in the same context.

Bad: Suppose f and g are linear functions. Then for some real numbers a and b , $f(x) = ax + b$ and $g(x) = ax + b$.

Good: Suppose f and g are linear functions. Then for some real numbers a, b, c , and d , $f(x) = ax + b$ and $g(x) = cx + d$.

9. Mathematical symbols like $=$, \exists (there exists), \forall (for all), and \Rightarrow (implies) should be used only in formulas, not on their own (or as verbs) in sentences.

Bad: The area of a rectangle is $=$ to the length times the width.

Good: The area of a rectangle is equal to the length times the width.

10. Important formulas should be included as a part of the complete sentence, but displayed on a separate line.

Bad: The solutions to the equation $ax^2 + bx + c = 0$ are given by the Quadratic Formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Good: The solutions to the equation

$$ax^2 + bx + c = 0$$

are given by the Quadratic Formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

11. Make sure that for every formula you explain either how you derived it or where it came from.
12. Don't just list formulas without any explanation – it makes them hard to read and it should be avoided.

Bad: $4x - 5 = 2x + 4$, $4x = 2x + 9$, $2x = 9$, $x = \frac{9}{2}$

Good: To solve the equation

$$4x - 5 = 2x + 4,$$

first add 5 to both sides of the equation to obtain

$$4x - 5 + 5 = 2x + 4 + 5$$

and

$$4x = 2x + 9.$$

Next subtract $2x$ from both sides to give

$$2x = 9.$$

Finally, dividing both sides by 2 gives the answer

$$x = \frac{9}{2}.$$

13. Use proper notation at all times.

Bad: The function f is defined by $f = x^3 + 2x - 1$.

Good: The function f is defined by $f(x) = x^3 + 2x - 1$.

14. Be aware that mathematics is case sensitive.

Bad: If $a = 3$, then $2A + 2 = 8$.

Good: If $a = 3$, then $2a + 2 = 8$.

15. Be specific and choose your words carefully. Try to avoid vague words like “it” for example.

16. Simplify whenever possible. Avoid large and complicated words when small words will get your point across.

17. Be cautious about words with double meanings or similar spelling. Make sure you are using the correct word in the right way.

Bad: Their is a function, $f(x)$ that has an interesting affect on x .

Good: There is a function, $f(x)$ that has an interesting effect on x .

18. Use the active, not passive, voice.

Bad: The function f was integrated over the infinite interval $[a, \infty)$ to obtain the result.

Good: To obtain the result integrate the function f over the infinite interval $[a, \infty)$

19. Thoroughly explain everything you do throughout a problem. This means you should not jump directly into a proof. Begin with an outline of what is to come.

20. There is a significant difference between the definite article “the” and the indefinite articles “a” and “an”. “The” implies uniqueness.

Bad: The antiderivative of x^3 is $\frac{1}{4}x^4$.

Good: An antiderivative of x^3 is $\frac{1}{4}x^4$.

21. Be clear and concise in your writing – use a format that is easy to follow and use transition phrases to guide the reader.

Bad: Let $f(x) = \frac{x}{(2x-1)^2}$. Then $f(x) = x(2x-1)^{-2}$,
so $f'(x) = 1(2x-1)^{-2} + x[-2(2x-1)^{-3}2] =$
 $\frac{1}{(2x-1)^2} - \frac{4x}{(2x-1)^3} = \frac{-2x-1}{(2x-1)^3}$.

Good: Let

$$f(x) = \frac{x}{(2x-1)^2}.$$

To find the derivative of f , either use the quotient rule or the product rule. To use the product rule write

$$f(x) = \frac{x}{(2x-1)^2} = x(2x-1)^{-2}$$

and differentiate to give

$$\begin{aligned} f'(x) &= 1(2x-1)^{-2} + x[-2(2x-1)^{-3} \cdot 2] \\ &= \frac{1}{(2x-1)^2} - \frac{4x}{(2x-1)^3}. \end{aligned}$$

The common denominator for the last two fractions is $(2x-1)^3$, so we can combine the fractions to get

$$\begin{aligned} f'(x) &= \frac{(2x-1) - 4x}{(2x-1)^3} \\ &= \frac{-2x-1}{(2x-1)^3}. \end{aligned}$$

22. Do not invent abbreviations that the reader is not familiar with.

23. Do not use the equal sign when you mean "which implies" (\Rightarrow).

Bad: $\cos^2 x - \cos x = 2 = \cos^2 x - \cos x - 2 = 0 = (\cos x + 1)(\cos x - 2) = 0 = \cos x = -1$ or 2 .

Good: The equation

$$\cos^2 x - \cos x = 2 \Rightarrow \cos^2 x - \cos x - 2 = 0$$

so factoring the left side gives

$$(\cos x + 1)(\cos x - 2) = 0 \Rightarrow \cos x = -1 \text{ or } \cos x = 2.$$

24. Do not give explanations to the right of equations. Avoid the two column method of proof.

Bad:	$x^2 - 3x = -2$	Solve the equation.
	$x^2 - 3x + 2 = 0$	Isolate 0 on the right.
	$(x-2)(x-1) = 0$	Factor.
	$x-2 = 0$ or $x-1 = 0$	If the product of two numbers is 0, at least one is 0.
	$x = 2$ or $x = 1$	Solve for x .

Good: To solve the equation

$$x^2 - 3x = -2$$

first isolate 0 on the right side, so

$$x^2 - 3x + 2 = 0.$$

Factoring the left hand side gives

$$(x - 2)(x - 1) = 0,$$

which is the case only when

$$x - 2 = 0 \text{ or } x - 1 = 0.$$

Now solve for x in both equations giving

$$x = 2 \text{ or } x = 1.$$

25. Clearly label all figures and diagrams so the reader understands exactly what they are seeing.
26. Don't be wordy, so omit unnecessary words.
27. Avoid starting a sentence with a word that has an "-ing" suffix.

Bad: Assuming $x = 1$ in the equation...

Good: When $x = 1$, the equation becomes...

28. Distinguish between *that* and *which*. Use *which*, preceded by a comma, when introducing nonessential information. Use *that*, never preceded with a comma, when introducing essential information.
29. **We vs. I**
Most readers consider the use of "I" to be pompous and inappropriate. It is at the very least irritating. The custom is to use the first person plural, or "we." Everything can be explained or described without the use of "I", so make an effort to avoid using it in your writing.
30. Avoid an isolated use of the word "this." Add a word to specify what "this" refers to.
31. **a or an**
Use *a* if the first letter of the following word begins with a consonant when pronounced. Use *an* if the first letter of the following word begins with a vowel sound when pronounced.

32. Avoid the use of contractions.
33. Try not to use words such as “obviously”, “clearly”, or “trivially”.
34. Make sure that your subject and verb belong together because subject and verb agreement is key.

Bad: The set of all prime numbers are infinite.

Good: The set of all prime numbers is infinite.

35. Equations always have two sides. Equations and expressions are different.

Bad: To evaluate the integral we first square the integrand so

$$\int_0^1 (x^2 + 1)^2 dx = \int_0^1 (x^4 + 2x^2 + 1) dx$$

and then using the power rule

$$= \frac{x^5}{5} + \frac{2x^3}{3} + x \Big|_0^1.$$

Good: To evaluate the integral we first equate the integrand so

$$\int_0^1 (x^2 + 1)^2 dx = \int_0^1 (x^4 + 2x^2 + 1) dx$$

and then using the power rule

$$\int_0^1 (x^2 + 1)^2 dx = \frac{x^5}{5} + \frac{2x^3}{3} + x \Big|_0^1.$$

Chapter 2

L^AT_EX Basics

2.1 Introduction

In this chapter we will describe the important steps of how to download and install the components for a complete system. We will then delve into how to begin using the L^AT_EX program and its most common, basic functions.

2.2 Downloading L^AT_EX

There are two programs needed to write documents in L^AT_EX. You will need an editor, we prefer the program WinEdt, and a version of L^AT_EX, here our preference is MiK_TE_X. The programs Ghostscript and Adobe Acrobat are also useful as viewers of L^AT_EX output documents. Downloads for each of these programs can be found on the internet using any basic search engine. The only program that has a fee associated with it is WinEdt.

You can download and use WinEdt at no cost for a 30 day trial period. After the 30 days have passed, the program will start bothering the user with pop-up reminders to register the program. If you register your version of WinEdt the messages will stop, if you don't they will just appear more frequently. The student fee is inexpensive and it is something that is good for life once you have made the initial investment.

The locations where the required software can be found are given in the following table. It provides the names of the files to download and the web site at which they can be found.

Program	File Name	Web Location (url)
Ghostscript	gs850w32.exe gsv46w32.exe	http://prdownloads.sourceforge.net/ghostscript/
MiK _T E _X	setup-2.4.1705.exe	http://www.miktex.org/setup.html
WinEdt	winedt54.exe	http://www.winedt.com/
Adobe Acrobat	AdbeRdr70_enu_full	http://www.adobe.com/products/acrobat/readstep2.html

2.3 Installing \LaTeX

In order for the components of these three programs to work properly, there is an order in which you will want to install them. The installation order is:

1. Adobe Acrobat
2. Ghostscript
3. MiK \TeX
4. WinEdt.

When installed in this order, WinEdt will find the exact locations of the other software and all the required buttons will be set in the editor. The following table shows, more specifically, how to correctly install each of the programs.

Software	Installation Instructions
Adobe Acrobat	Follow the directions on the website, and then run <code>AdbERdr70_enu_full</code> . Select all defaults.
Ghostscript	Select all defaults. <ol style="list-style-type: none"> 1. Run <code>gs850w32.exe</code>. 2. Run <code>gsv46w32.exe</code>.
Miktex	<ol style="list-style-type: none"> 1. Extract all files by unzipping the zip file into an empty folder. See www.winzip.com for Winzip for unzipping files. 2. Run <code>setup-2.4.1705.exe</code>. Select download only and then all of the defaults. 3. Run <code>setup-2.4.1705.exe</code> again. This time select install and then all of the defaults.
WinEdt	Run <code>winedt54.exe</code> . Select all defaults.

2.3.1 Testing the Installation

In order to check and see if you have installed all of the components correctly, follow the set of directions to produce a sample document.

1. Open the program WinEdt and create a new document by choosing File+New or using `Ctrl+N`.

2. In the new blank document, type the following exactly as it appears.

```
\documentclass{book}

\begin{document}
\begin{center}
{\Large \bf Testing \LaTeX\ Installation...}
\end{center}
\end{document}
```

3. Compile this sample document by clicking on the small picture of a brown bear in the tool bar at the top of the window. You will see a black box appear and it will scroll through some information. It should automatically open a new window with the correct output. If everything is working properly, you will see

Testing \LaTeX Installation...

2.4 Creating a New Document

Begin by opening the editor in which you will type, in our context, the program you want to open is WinEdt. You should find one link to the program on your desktop and, possibly, another pinned to the start menu. Find one of the WinEdt links and click on it – the program should open. Go to the File menu and click on New (or Ctrl+N), which will open a new and blank document.

Every time it is necessary to start a new document, there are a few elements that are required. In the *preamble* (the part before the actual document begins), the first element that must be defined is the document type. This is done by specifying the command `\documentclass{...}` as the first line. In between the curly braces is where the type of document is stated. The different document types determine the layout of the pages. Two common document classes are *book* and *article*. Just so you know, if you use the book document class, the page numbers will be in the upper corners and they will alternate, the even page numbers on the left side and the odd page numbers on the right side.

The next item to be defined in the preamble of the document is which packages must be loaded. The way to specify which packages the program should use, you type the command `\usepackage{...}` with the name of the desired package inside of the curly braces. Another thing to know is that there are certain basic packages that are good to have for every document. It would be wise and recommended to use the following basic format when you create each new document.

```

\documentclass{book}
\usepackage[dvips]{graphicx}
\usepackage{graphics,amsfonts,amsmath,amsbsy,amssymb}

\begin{document}

\end{document}

```

By including the packages seen above you have activated the fonts and symbols commonly used in most documents. Note that anything outside of the begin and end document will not be displayed in the output.

Each and every time you create a new document, you must save it. When you decide upon a name for your file, I recommend avoiding spaces in your file name. If you can, it is best to make it a single word, you can use numbers if you like. Every subsequent time you compile your document (I will later explain what compiling is and what it does) and view the output, the program will automatically save your file for you. What is compiling a document, you may ask, so here it is.

2.5 Compiling a Document

As you type in the WinEdt program, you are merely placing the information into an editor. The commands that you are typing are obviously not what you plan on printing out and handing in. To see the actual output, one must compile the document, and it will then be viewed using one of the programs you downloaded. You can compile the document by clicking on a few different buttons in the toolbar, or typing their associated commands. L^AT_EX is a programming language and when a file is compiled the commands are interpreted and the requested action is taken. All commands in L^AT_EX begin with the backslash (\) followed by the command name.

If you just want to see whether or not your document works, you can either click on the button that reads L^AT_EX. The command for this button is Shift+Ctrl+L. When you click on this buttons, the brains of the L^AT_EX program create a .dvi file, which must be manually opened. You can view the .dvi file by clicking on the button that shows a magnifying glass and displays DVI on the button or the one with a picture of a ghost wearing really large glasses. The commands for these two buttons are Shift+Ctrl+V and Shift+Ctrl+G respectively.

You can compile your document and have the output automatically open by clicking on the button on which there is a picture of a little bear. This shortcut will not always work, depending on the complexity of your document. The default program used to view documents when using this button might not have the ability to show some of the more advanced output. The solution to this dilemma is simple – make a PDF file.

2.5.1 Creating PDF Files

As discussed above, all documents can be compiled using the \LaTeX button, which creates a .dvi file. In order to convert this file into a PDF file, you must click on the button “dvi \leftrightarrow ps”. You then click the button “ps \leftrightarrow pdf”. As long as you have installed the program Adobe Acrobat Reader, the PDF file should automatically open. PDF files are common and most computers have Adobe Acrobat Reader, so creating a PDF file version of a document can be useful if you must open and view or print a document on a computer that does not have \LaTeX .

2.6 Understanding Your Files

Something to be aware of is that every time you compile your document and the program saves your file, it does not only save the .tex file, but it will save a variety of other files as well.

File Type	Description
.dvi	Device Independent File. This is the main result of a \LaTeX compile run. You can look at its content with a DVI previewer program or you can send it to a printer with dvips or a similar application.
.log	Gives a detailed account of what happened during the last compiler run.
.aux	A file that transports information from one compiler run to the next. The .aux file is used to store information associated with cross-references.
.tex	The \LaTeX or \TeX input file. Can be compiled using \LaTeX .
.tex.bak	A backup version of your .tex file.

2.7 Plain Text, Math Text, and Symbols

\LaTeX is an extension of the program \TeX . The designers of \LaTeX have added numerous commands and packages to \TeX . In \LaTeX there are generally two modes for typing. You will either be typing in “text mode” or “math mode”, as I will refer to them. Similarly, later we will discuss the different environments, some of which will be text environments and others math environments.

2.7.1 Text Mode

When you first begin any document, you are in the regular “text mode”. In this mode you can type as you would in any word processing program. \LaTeX will determine for you how to space your document, so all you have to worry about is placing the correct words on the page in the right order. One thing to note, about writing plain text, is that every time you leave a blank line between blocks of text, \LaTeX will interpret that as two separate paragraphs, so the second will be indented. If you ever want to do that, but tell \LaTeX to not indent the second paragraph, just place `\noindent` (followed by a space to

show that you are ending the command) in front of the first word in your second paragraph.

Two other handy commands when writing text are those which allow you to put text onto the next line or even the next page. You can always leave a blank line between paragraphs and use the noindent command I just mentioned, or you can type `\newline` between the two chunks of text. This will move your second paragraph to a new line, without indenting. Also, if you come across a point in your document where you feel that the L^AT_EX program has broken up text (between pages) in a bad place, you can precede your text with the command `\newpage` to push it all onto the next page.

2.7.2 Math Mode

When you want to insert math into your document, you use “math mode” to do so. The important character to remember in math mode is the dollar sign (`$`), so just remember that math is money. Whatever you enclose with single dollar signs on either side will be read by L^AT_EX as inline math. For larger mathematical expressions, inline math will not do them justice and you should use the centered math mode. To center mathematical expressions, the only difference is an extra dollar sign on either side. So, if you enclose your expression in single dollar signs it will be placed inline with your text and when you enclose an expression in double dollar signs it will be centered. Centered math makes for a beautiful display of important or key points in your argument.

Remember that many of the environments are initially in text mode, but you can include mathematical terms or expressions inside of them, just enclose the items in dollar signs.

2.7.3 Special Characters and Symbols

There are certain characters that are used in the L^AT_EX program as commands, therefore you will find errors in your document if you try to type them regularly (to be seen in the output). The symbols before which you must place a backslash (or a defining command) if you want them to appear in the output are,

$$\$ \% \& * ^ - \ \{ \}$$

and these are only a few of the symbols that exist in L^AT_EX.

You do not have to memorize every command for every symbol or action that you want to use because of the wonderful tool bar of L^AT_EX symbols that is part of the WinEdt environment. In case you have not yet discovered this tool bar, it can be viewed by clicking on the button (in the array of document tools at the top of the WinEdt window) that has a sigma (Σ) on it. Included in the thirteen tabs of that tool bar is just about any command or symbol that you will ever need. See Appendix A.

The neat thing about this feature is that you can find the command for any symbol in that tool bar, but you will soon find that the commands you most frequently use will become easy for you to remember with time. The more you

use this program and the more practice you have typing commands, the faster and more comfortable you will get with everything.

2.7.4 Using Comments

Although it is recommended that you compile your document after every few sentences or mathematical expressions that you type, people still experience errors. If you get too far ahead of yourself and forget to compile your document, you may not know where to find the errors that need to be fixed. There is a method that works well in this situation.

You can comment out as much text as you think *could* be problematic. This is done by first highlighting everything that you think could possibly contain the error. Next, right click on that selection you just made and choose the “Insert Comment” option. This places a percentage sign (%) at the beginning of every line you highlighted, and they tell \LaTeX to ignore the line. Any line or lines can be commented out manually by placing percentage signs at the beginning of the line, but this can be a tedious process if you have a lot of information to comment out.

Make sure that your document compiles correctly with this section now commented out, and then slowly work through to find your error. If you go line by line, compiling the document after you remove the comments one at a time, you will eventually come across the problem. When you want to remove the rest of the comments, you can simply delete the percentage signs. If you still have many lines with comments, just highlight the commented lines, right click on the selection, and choose “Remove Comment” from the options.

It is important to note that when searching for errors using comments you can’t always remove one comment line at a time if environments are involved. For example, if you comment out the `\begin{itemize}` in the `itemize` environment, you will encounter an error because \LaTeX will be searching for the `\end{itemize}` and at least one `\item` in between them. So, use caution when using comments.

Commenting out sections of a document can be used for other purposes as well. If you aren’t sure that you want to delete something yet, but you don’t want to view it every time you compile your document, just highlight the section and comment it out. Sometimes if you are including graphics and they take a long time to compile every time, it is sometimes easier to make sure they work and then comment them out until you are finished and want to print the final output. It just comes down to the fact that inserting and removing comments in your document is a useful tool in \LaTeX .

2.8 Environments

There are a variety of environments that do many different things. As mentioned earlier, certain environments are in “text mode” and others are already in “math mode” so it is important to know which you are dealing with. If an environment

is already in “math mode” then you do not need to use dollar signs inside of it. On the other hand, if you want to insert mathematical variables or statements inside of a text environment, you must include dollar signs around those terms.

The two most common text environments used for a homework assignment would include those which create a list with bullets in front of the items, known as `itemize`, or a numbered list of items, which is called `enumerate`. You begin and end each of the environments, and in the middle all you must do is define where the new items begin. For example, if we wanted to create a numbered list of five Greek letters, we can use

```
\begin{enumerate}
\item  $\alpha$ 
\item  $\beta$ 
\item  $\gamma$ 
\item  $\delta$ 
\item  $\epsilon$ 
\end{enumerate}
```

which would give the output,

1. α
2. β
3. γ
4. δ
5. ϵ

Another example is that of a list with bullets in front of each item. For this case we use

```
\begin{itemize}
\item  $x < 5$ 
\item  $x = 5$ 
\item  $x > 5$ 
\end{itemize}
```

for which the output appears as

- $x < 5$
- $x = 5$
- $x > 5$

2.9 Tables and Arrays

Tables and arrays are almost the same thing, the only difference is that tables are in text mode and arrays are in math mode. The best part of these two environments is that they can be modified and personalized very easily. You can include as many or as few of the lines that are needed in a table, using the tabular environment. An example of a table is

x	$f(x) = x^2$	y
-2	$(-2)^2$	4
3	$(3)^2$	9

and the commands to produce it are shown below.

```
\begin{center}
\begin{tabular}{r|c|c}
% after \\: \hline or \cline{col1-col2} \cline{col3-col4} ...
$x$ & $f(x)=x^2$ & $y$ \\
\hline
$-2$ & $(-2)^2$ & 4 \\
3 & $(3)^2$ & 9 \\
\end{tabular}
\end{center}
```

The specification “r|c|c” indicates how the three columns are justified. Individual columns can be left justified by specifying “l”, right justified by specifying “r”, or centered using “c”. To separate columns with vertical lines place a | between each column specification. When entering the data for individual columns, the columns are separated with the ampersand & . The number of rows can vary with each row ending with \\. Rows can be separated with horizontal lines using \hline .

The entries of an array are already in math mode, so dollar signs are not needed. For example, to draw a 2×2 matrix, that includes the entries a , b , c , and d , the commands are

```
$$
\left(
\begin{array}{cc}
a & b \\
c & d
\end{array}
\right)
$$
```

and the output appears as

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

2.10 Sized to Fit

2.10.1 Making Things Grow

Many mathematical expressions are surrounded by parentheses or braces. The surrounding symbols should grow with the expression inside to maintain the aesthetics of the expression. To have the symbols surrounding an expression grow precede the left enclosing character with `\left` and the right enclosing character with `\right` as in

```
\left( .... \right)
```

For example, to set

$$\left(\frac{x}{x^2 - 2x + 1}, \frac{x + 1}{x^2 + 3x - 4} \right)$$

we can use

```
$$
\left(\frac{x}{x^2-2x+1}, \frac{x+1}{x^2+3x-4}\right)
$$
```

Braces must be preceded with a `\` as in `\left\{...\right\}`, since the brace has other uses in the language L^AT_EX.

If a brace or parenthesis, etc. is required only on the left side, as in

$$\left\{ \begin{array}{l} 0, \text{ if } x = 1, \\ 5, \text{ if } x > 1. \end{array} \right.$$

end the right with a period. The exact commands are

```
$$
\left\{\begin{array}{cc}
0, & \text{\textmd{if } } x=1, \\
5, & \text{\textmd{if } } x>1.
\end{array}\right.
\right.
$$
```

2.10.2 Changing Text Size

Not only can the size of symbols be changed, but text can grow or shrink as well. Changing the size of a font is useful in showing emphasis and in creating headings. There are several easy font changes that can be used for making larger or smaller fonts. For larger fonts try `\large` or `\Large` and for smaller font sizes try `\small` or `\footnotesize`. To change the font size of specific text use `{\size text here}`. Enclosing the font change in braces restricts the change to the block of text in the braces. The command `\normalise` sets the font size to the default size.

Display	Font Change
L ^A T _E X	Default font size.
L ^A T _E X	<code>\small</code>
L ^A T _E X	<code>\footnotesize</code>
L ^A T _E X	<code>\large</code>
L ^A T _E X	<code>\Large</code>

2.11 Extra Space

One neat aspect of L^AT_EX is that you can format your document to look any way that you want it to look. If you ever need to shift anything up, down, left, or right, you just have to space it appropriately. You can move things to the right or left using horizontal space for which the command is `\hspace{}` and inside of the curly braces is where you specify how much space you want. Space can be specified in different units as well, use pt for point and in for inches. For example, if you had the expression

$$3x^2yz$$

and you added 10pt of horizontal space between the y and the z , the result would be

$$3x^2y \quad z$$

Similarly, you can add vertical space by using the command `\vspace{}` to increase the vertical distance between items. This can be seen in the following text sample:

Here we have words
on two lines.

When we add half an inch of vertical space between the two lines, the result is the following.

Here we have words

on two lines.

One final thought on adding space is that you can also use negative parameters to move things left or up. For example, if we took the following text

NICELY
SPACED

for which the commands are

```
\begin{center}
NICELY\\
SPACED
\end{center}
```

and we insert -15pt of vertical space, the commands become

```

\begin{center}
NOT\
\vspace{-5pt}
NICELY\
\vspace{-15pt}
SPACED
\end{center}

```

and the output then appears as

NOT
SPACED
NICELY

This is a perfect example for the use of negative parameters when considering altering vertical and horizontal space.

2.12 Applying the Basics

Now that we have covered some of the most common features of L^AT_EX we will show a few examples of how it can be used to set a document. In the following three examples, we will show you exactly what must be typed and then what the output will look like. Some of the elements in these examples use more advanced features of L^AT_EX that will be described later in this document or can be found in Dr. DeFranza's manual on writing mathematics. [2]

In the following example, you will see the use of the enumerate environment, fractions, integrals, square root signs, displayed math, and a number of simple commands. The L^AT_EX is shown, followed by the output.

Commands:

```

\documentclass{book}
\usepackage{amsmath}

\begin{document}
\centerline{\large{\LaTeX\ Example 1}}
\begin{enumerate}
\item{$$\int \frac{u^ndu}{\sqrt{a+bu}} = -\frac{2u^n \sqrt{a+bu}}{b(2n+1)} - \frac{2na}{b(2n+1)} \int \frac{u^{n-1}du}{\sqrt{a+bu}}$$}
\item{
$$
\int \sin\{au\}\cos\{bu\}du =
-\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C
$$ }
\item{
$$ \sum_{k=1}^n k = 1 + 2 + \cdots + (n-1) + n = \frac{n(n+1)}{2} $$ }

```

```

\item{
$$
\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{1 - \frac{1}{2}} = 2
$$
}
\item{
If  $ax^2+bx+c=0$ , then  $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ .
}
\end{enumerate}
\end{document}

```

Output:L^AT_EX Example 1

1.
$$\int \frac{u^n du}{\sqrt{a+bu}} = -\frac{2u^n \sqrt{a+bu}}{b(2n+1)} - \frac{2na}{b(2n+1)} \int \frac{u^{n-1} du}{\sqrt{a+bu}}$$
2.
$$\int \sin au \cos bu du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$$
3.
$$\sum_{k=1}^n k = 1 + 2 + \cdots + (n-1) + n = \frac{n(n+1)}{2}$$
4.
$$\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{1 - \frac{1}{2}} = 2$$
5. If $ax^2 + bx + c = 0$, then
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

In the second example you will see the use of summation signs, making things grow, an array, and a many other simple commands. The L^AT_EX is shown, followed by the output.

Commands:

```

\documentclass{book}
\usepackage{amsmath}

\begin{document}
\centerline{\large{\LaTeX\ Example 2}}
\begin{enumerate}
\item{

```

```

\begin{align*}
\sum_{n=1}^{\infty} \frac{1}{2^n} &= \frac{1}{1-\frac{1}{2}} \setminus \\
&= 2 \\
\end{align*}
\item{The midpoint of the line segment connecting the two points
 $P_1(x_1, y_1)$  \text{ and }  $P_2(x_2, y_2)$  is the point
}
\text{Midpoint:} \quad \left( \frac{x_1+x_2}{2}, \right. \\
\left. \frac{y_1+y_2}{2} \right). \\
}
\item{Let  $f$  be the piecewise defined function given by
}
f(x) = \left\{ \begin{array}{lll}
x^2 - 2x + 1 & \text{for } & x < -2 \\
x + 1 & \text{for } & -2 \leq x < 3 \\
\sqrt{x} & \text{for } & x \geq 3.
\end{array} \right. \\
\right. \\
}
\item{
}
\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{u^2} \\
\ln \left| \frac{a+bu}{u} \right| + C \\
}
\item{
}
(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2}x^{n-2}y^2 + \cdots + \\
\left( \begin{array}{l} n \\ k \end{array} \right) x^{n-k}y^k + \cdots + nxy^{n-1} + y^n \\
}
\end{enumerate}
\end{document}

```

Output:L^AT_EX Example 2

1.

$$\begin{aligned}\sum_{n=1}^{\infty} \frac{1}{2^n} &= \frac{1}{1 - \frac{1}{2}} \\ &= 2\end{aligned}$$

2. The midpoint of the line segment connecting the two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is the point

$$\text{Midpoint: } \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

3. Let f be the piecewise defined function given by

$$f(x) = \begin{cases} x^2 - 2x + 1 & \text{for } x < -2 \\ x + 1 & \text{for } -2 \leq x < 3 \\ \sqrt{x} & \text{for } x \geq 3. \end{cases}$$

4.

$$\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{u^2} \ln \left| \frac{a+bu}{u} \right| + C$$

5.

$$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2}x^{n-2}y^2 + \cdots + \binom{n}{k}x^{n-k}y^k + \cdots + nxy^{n-1} + y^n$$

The final example is the most complex, in which we use tables, footnotes, and inserting graphics. Some of these features will be described in later chapters of this document, but those that are not can be found in any of the numerous L^AT_EX manuals that exist. The L^AT_EX is shown, followed by the output.

Commands:

```
\documentclass{book}
\usepackage{graphics,amsfonts,amsmath,amsbsy,amssymb}

\begin{document}
\centerline{\large{\LaTeX\ Example 3}}
```

The data in the table for cumulative HIV infections and deaths due to aids in millions worldwide indicate that in the year 2000 someone was infected every 20 seconds.\footnote{Vital Signs 2001, The Trends That Are Shaping Our Future, W.W. Norton and Company, New York London.}

```

%\begin{center}
%\begin{table}
%\begin{tabular}{ccc}
%Year& HIV Infections& Aids Deaths\\
%\hline
%$1980$$0.1$$0.0$\\
%$1985$$2.4$$0.2$\\
%$1990$$10.0$$1.7$\\
%$1995$$29.8$$8.2$\\
%$2000$$57.9$$21.8$\\
%\end{tabular}
%\end{table}
%\end{center}

```

```

\vspace{10pt}
\begin{center}
%\begin{table}
\begin{tabular}{ccc|ccc}
Year& HIV Infections& Aids Deaths& Year& HIV Infections& AIDS
Deaths\\
\hline
$1980$$0.1$$0.0$$1991$$12.8$$2.4$\\
$1981$$0.3$$0.0$$1992$$16.1$$3.3$\\
$1982$$0.7$$0.0$$1993$$20.1$$4.7$\\
$1983$$1.2$$0.0$$1994$$24.5$$6.2$\\
$1984$$1.7$$0.1$$1995$$29.8$$8.2$\\
$1985$$2.4$$0.2$$1996$$35.3$$10.6$\\
$1986$$3.4$$0.3$$1997$$40.9$$13.2$\\
$1987$$4.5$$0.5$$1998$$46.6$$15.9$\\
$1988$$5.9$$0.8$$1999$$52.6$$18.8$\\
$1989$$7.8$$1.2$$2000$$57.9$$21.8$\\
$1990$$10.0$$1.7$\\
\end{tabular}
%\end{table}
\end{center}

```

HIV is the virus that causes AIDS. In the year 2000 approximately 5.3 million people contracted the virus and since 1980 the virus has killed approximately 22 million people world wide. The table gives both HIV infections and AIDS deaths as functions of time. Plotting the data points and connecting successive points by straight line segments shows clearly the increasing trends in HIV infections and AIDS deaths.

```

\begin{center}

```

```
\resizebox{2.5in}{!}{\includegraphics{newcc1.eps}}
\end{center}
```

The data points in the table or the curves in the figure can be used to represent functions $H(x)$ and $A(x)$ for HIV infections and AIDS deaths. Then the function representing cumulative survivors of AIDS, $S(x)$, will be defined by

$$S(x) = H(x) - A(x).$$

In the table this corresponds to subtracting for each year the number of AIDS deaths from the number of HIV infections or subtracting the corresponding y -coordinates from points on the graphs as shown in the figure.

```
\begin{center}
\resizebox{2.5in}{!}{\includegraphics{newcc2.eps}}
\end{center}
```

A function describing the cumulative death rates as a percentage of the cumulative number of infections is

$$R(x) = 100 * \frac{A(x)}{H(x)}$$

and plotting the data shows the steady increase in the death rate. In calculus instantaneous rates of change are calculated using the notion of the *derivative* of a function and is the concept that allows the application of calculus to a wide range of areas.

```
\begin{center}
\resizebox{1.75in}{!}{\includegraphics{newcc3.eps}}
\end{center}
```

The two functions $S(x)$ and $R(x)$ are examples of combinations of other functions, or examples of constructing new functions from old functions.

```
\end{document}
```

Output:

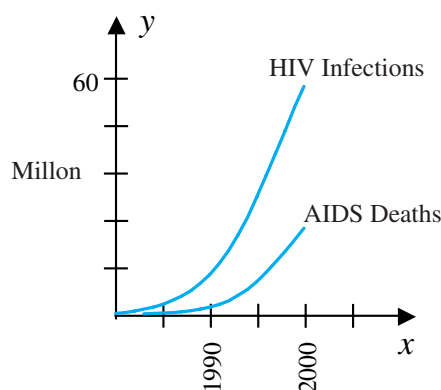
L^AT_EX Example 3

The data in the table for cumulative HIV infections and deaths due to aids in millions worldwide indicate that in the year 2000 someone was infected every 20 seconds.¹

¹Vital Signs 2001, The Trends That Are Shaping Our Future, W.W. Norton and Company, New York London.

Year	HIV Infections	Aids Deaths	Year	HIV Infections	AIDS Deaths
1980	0.1	0.0	1991	12.8	2.4
1981	0.3	0.0	1992	16.1	3.3
1982	0.7	0.0	1993	20.1	4.7
1983	1.2	0.0	1994	24.5	6.2
1984	1.7	0.1	1995	29.8	8.2
1985	2.4	0.2	1996	35.3	10.6
1986	3.4	0.3	1997	40.9	13.2
1987	4.5	0.5	1998	46.6	15.9
1988	5.9	0.8	1999	52.6	18.8
1989	7.8	1.2	2000	57.9	21.8
1990	10.0	1.7			

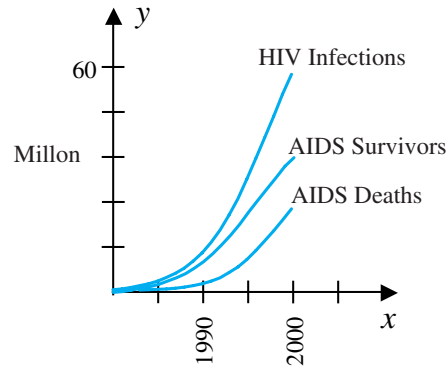
HIV is the virus that causes AIDS. In the year 2000 approximately 5.3 million people contracted the virus and since 1980 the virus has killed approximately 22 million people world wide. The table gives both HIV infections and AIDS deaths as functions of time. Plotting the data points and connecting successive points by straight line segments shows clearly the increasing trends in HIV infections and AIDS deaths.



The data points in the table or the curves in the figure can be used to represent functions $H(x)$ and $A(x)$ for HIV infections and AIDS deaths. Then the function representing cumulative survivors of AIDS, $S(x)$, will be defined by

$$S(x) = H(x) - A(x).$$

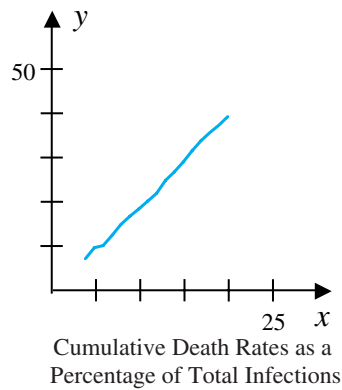
In the table this corresponds to subtracting for each year the number of AIDS deaths from the number of HIV infections or subtracting the corresponding y -coordinates from points on the graphs as shown in the figure.



A function describing the cumulative death rates as a percentage of the cumulative number of infections is

$$R(x) = 100 * \frac{A(x)}{H(x)}$$

and plotting the data shows the steady increase in the death rate. In calculus instantaneous rates of change are calculated using the notion of the *derivative* of a function and is the concept that allows the application of calculus to a wide range of areas.



The two functions $S(x)$ and $R(x)$ are examples of combinations of other functions, or examples of constructing new functions from old functions.

Chapter 3

Advanced L^AT_EX

3.1 Introduction

In the previous chapter, the basic commands needed to compose an average homework assignment were presented. In this chapter, our goal is to offer a few more of the features of L^AT_EX that can be used for writing a larger document, like an honors thesis for example.

3.2 Major Documents

You will quickly realize that L^AT_EX documents can get rather lengthy once all the text, mathematical equations, and other items are included. So, when you want to compose a document like an honors thesis, the best thing to do is create a master L^AT_EX file and other individual files for each section or chapter you need, depending on their length.

In your master document, you will then insert the other files you want to include, in any order. The important thing to know is that your begin and end document commands and any preamble commands will **only** be in your master document. You will create the other files, and in those you can include the begin and end document commands and any needed preamble commands, but they must be commented out when you compile the master document.

The command needed to include other L^AT_EX files in your master document is

```
\input{}
```

and the name of the L^AT_EX file (including the .tex at the end of it) goes in between the curly braces. The other possible parts of a master document will be described in the following subsections and a sample master document will be included at the end of the section.

3.2.1 Introductory Elements

To begin a large document, you most likely want or need a title page. For this, you only need to define two parameters and you can add any additional material you want to appear on your title page, such as a description or some graphics. The two parameters that must be provided are the title and the author. As an example, I will show the L^AT_EX commands for the title page of this document, so that you can see how it was created.

```
\title{\LaTeX\ and Writing Mathematics\\}
\author{Katri A. Nattie\\
{\small Supervisor: Jim DeFranza}\\
St. Lawrence University}
\maketitle
```

Another nice feature of L^AT_EX is that if you want to include a table of contents at the beginning of your document, the command is trivial, and L^AT_EX does all the hard work for you. The only command you need to insert into your document is

```
\tableofcontents
```

and it must be placed where you want it to appear in the final document. Any chapters or sections (described in the following section) that appear in your document before the table of contents will not automatically be included in it, but they may be defined and added if you want. The command needed to include extraneous items in the table of contents is

```
\addcontentsline{toc}{}{}
```

and in the first set of empty curly braces you must identify if you want it included as a chapter, section, or subsection. The second empty set of curly braces is where you include the text you want to appear as a table of contents item. For example, if we wanted to add a section named “Testing” into the table of contents in a document, the command is

```
\addcontentsline{toc}{section}{Testing}
```

Examples of items that you might want to include, like we just did, would be a Preface or a page of Acknowledgements. The title page would obviously come before the rest of the document, but it is not necessary to cite your title page in the table of contents.

3.2.2 Chapters, Sections, Subsections, and Subsubsections

Creating different divisions in a document is possibly one of the easiest things to do. If you are writing a large document, the commands are shown in the table below.

Command	Purpose
<code>\chapter{Title goes here...}</code>	New Chapter
<code>\section{Title goes here...}</code>	New Section
<code>\subsection{Title goes here...}</code>	New Sub-section
<code>\subsubsection{Title goes here...}</code>	New Sub-sub-section

For example, if you wanted to have a chapter called “Making Chapters”, the command is

```
\chapter{Making Chapters}
```

and it appears in your document in large font as a chapter title. Sections have a smaller font size and for subsections it is just a little smaller. The neat part is that no matter what order you put them in and no matter how many times you move them around, \LaTeX will number everything in the order you place the different parts.

3.2.3 Bibliography

The bibliography provides the references used to support your work. Supporting material for your statements adds credibility and is an essential component of mathematical writing. As a general rule citations should be provided for:

- All direct quotations from another author.
- All substantial information taken from another source.

The format of bibliography entries will depend on the selection of a style provided by \LaTeX . For a reference to a document found on the Internet a standard form is:

Doe, J, *Title of the Document*, url, date of last revision.

There is one slight difference to note when creating a bibliography file and adding it into your master document. When you save your \TeX file that includes your bibliographical information, save it with `.bib` on the end of the file name, not `.tex` like usual.

Bibliography Entries

The entries for a bibliography are stored in files with extensions `.bib`. A typical entry for a book is:

```
@BOOK{GP,
AUTHOR={George Polya},
TITLE={How To Solve It},
EDITION={Second},
YEAR={1985},
PUBLISHER={Princeton University Press},
ADDRESS={Princeton, New Jersey}}
```

where the first field `GP` is the reference that associates this entry with a citation in a document. For example, to refer to this entry in a document the command is:

```
\cite{GP}
```

An entry for an article has the form:

```
@ARTICLE{KN,
  AUTHOR={Donald Knuth},
  JOURNAL={Bulletin AMS},
  TITLE={Mathematical Typography},
  YEAR={1979},
  VOLUME={1},
  NUMBER={2},
  PAGES={337-372} }
```

The actual form the entries take in the bibliography is determined by the style specified in the `\bibliographstyle{.}` command. See the sample master document at the end of this chapter.

3.3 Appendices

The situation often arises where certain information is important as background material, alternative approaches to certain parts of the main material, or just additional information that is too much to include in the text proper. The solution is to include the information in an Appendix. The information is then included but does not interrupt the flow of ideas in the main part of the text. Appendices appear at the end of the document and are usually sequentially ordered as Appendix A, Appendix B, etc. In L^AT_EX the start of the appendices is indicated with the command

```
\appendix
```

and the beginning of each appendix is given by

```
\chapter{Title of the appendix goes here...}
```

followed by the material for the appendix. This can be in a separate file and included using `\input{name}`.

3.4 Mini Pages

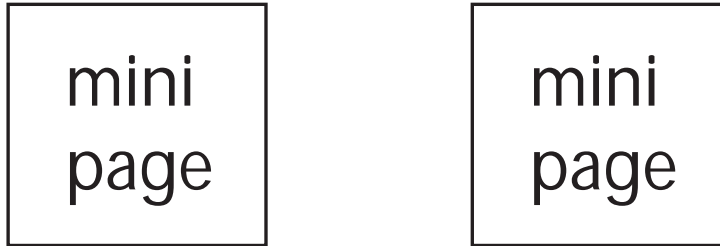
To set several figures or blocks of text, etc., side by side the *minipage* environment can be used. The *minipage* environment constructs a paragraph box using the syntax

```
\begin{minipage}[pos]{width}
  text
\end{minipage}
```

The width of the box is *width* and the vertical positioning of the box with respect to the text baseline is controlled by the one letter parameter *pos* ([c], [t], [b]). For example,

```
\begin{figure}[h]
\begin{minipage}[t]{0.5\linewidth}
\resizebox{1.5in}{!}{\includegraphics{minipage.eps}}
\end{minipage}
\begin{minipage}[t]{0.5\linewidth}
\resizebox{1.5in}{!}{\includegraphics{minipage.eps}}
\end{minipage}
\end{figure}
```

centers two boxes side by side allowing half the width of a line for each minipage.



3.5 Writing New Commands

Anything that is used repeatedly in a document can be expressed with its own command. This can be as simple as the same text appearing in different places to much more complicated structures that are numbered sequentially. The following example shows how a standardized example environment can be created including automatic numbering of the example. The definition is given in the preamble and the command can then be used anywhere in the document. The numbering is accomplished by defining a new counter variable, in the example it is `exnum`.

```

\newcounter{exnum}

\newcommand{\example}[1]{
\addtocounter{exnum}{1}
\noindent {\bf E{\scriptsize XAMPLE} \arabic{exnum}:} #1
\vspace{10pt}
}

```

For example, if in a document we use

```
\example{This is the first example.}
```

```
\example{This is the second example.}
```

the output produced is

EXAMPLE 1: This is the first example.

EXAMPLE 2: This is the second example.

The possibilities are endless.

3.6 Sample Master Document

Here is the master document for this document.

```

\documentclass{book}
\usepackage[dvips]{graphicx}
\usepackage{graphics,amsfonts,amsmath,amsbsy,amssymb}
\usepackage{verbatim}
\usepackage{psfrag}
\usepackage{cwpuzzle,crosswrd}
\usepackage{onepage}
\usepackage[active]{srcltx}
\usepackage{amsthm}
\usepackage{pifont}
\usepackage{enumerate}
\usepackage{boxedminipage}
\usepackage{color}
\usepackage{xspace}
\usepackage{setspace}

\setlength{\oddsidemargin}{\evensidemargin}

\usepackage{ifthen}

```

```

\newcommand{\mc}[1]{%
\ifthenelse {\isodd{\value{page}}}{\reversemarginpar}{}%
\marginpar{\small%
#1%
}%
\normalmarginpar%
}%

\newcounter{exnum}

\newcommand{\example}[1]{
\addtocounter{exnum}{1}
\noindent {\bf E{\scriptsize XAMPLE} \arabic{exnum}:} #1
\vspace{10pt}
}

\newcommand{\gb}[2]{
%\noindent\hspace{10pt}
\begin{tabbing}
\hspace{10pt} \= \hspace{30pt} \= \hspace{20pt} \= \kill
  \> Bad: \> \parbox[t]{3in}{#1} \\
  \> Good:\> \parbox[t]{3in}{#2}
\end{tabbing}
}

\begin{document}
\title{
\LaTeX\ and Writing Mathematics\\}
\author{Katri A. Nattie\\
{\small Faculty Advisor: Jim DeFranza}\\}
\\
{\small Committee Member: Michael Schuckers}\\}
{\small Committee Member: Daniel Gagliardi}\\}
\\
St. Lawrence University}
\maketitle
\pagenumbering
\newpage

\bibliographystyle{plain}
\pagenumbering{roman}
\input{Acknowledgements.tex}
\addcontentsline{toc}{section}{Acknowledgements}
\newpage
\addcontentsline{toc}{section}{Preface}
\input{Preface.tex}
\nocite{*}

```

```
\tableofcontents
\chapter{Introduction to Writing Mathematics}
\pagenumbering{arabic}
\input{Intro.tex}
\chapter{\LaTeX Basics}
\input{TexBasics.tex}
\chapter{Advanced \LaTeX}
\input{AdvancedTex.tex}
\chapter{Writing Proofs}
\input{ProofExamples.tex}
\chapter{Using Graphics}
\input{GraphicsGOOD.tex}
\chapter{Project Analysis}
\input{Summary.tex}
\appendix
\chapter{\LaTeX Symbols}
\input{mysymbols.tex}
\bibliography{Bibliography}
\addcontentsline{toc}{section}{Bibliography}
\end{document}
```

Chapter 4

Writing Proofs

4.1 Introduction

This chapter begins with a description of George Polya's insights regarding problem solving. In this chapter, we describe these principles of problem solving and then employ them in the context of technique of proof. Four of the most common and important types of proofs are presented with appropriate examples to clearly illustrate each method. Although graphics are created and inserted in the proof examples, Chapter 5 is where this process is described, so go there for help with setting figures.

4.2 Techniques of Problem Solving

George Polya was one of the early writers on problem solving, specifically describing basic principles of problem solving that are often used unconsciously by mathematicians. His very straightforward approach results in a clear set of simple principles that can be employed by the novice and expert alike.

According to George Polya, the most important thing to consider in problem solving is “Trying to understand not only the solution of this or that problem but also the motives and procedures of the solution, and trying to explain these motives and procedures to others” [6].

In Polya's book, How To Solve It, he describes a process that can be used when solving any problem (which can be done unconsciously by some people). Some of the important things to consider are given in the following list:

- Identify the quantity to find or statement to prove.
- Isolate the data in the problem.
- Find an expression that relates the data given and the quantity to find.
- Draw a picture.

- Use previous knowledge.
- Introduce an auxiliary problem.
- Check your solutions.
- Use all the conditions of the problem.
- Consider different cases.
- Look for patterns.
- Describe the problem statement in your own words.
- Find an equation that relates the data given and the quantity to find.
- Find a function that relates the data given and the quantity to find.
- Use all the data.

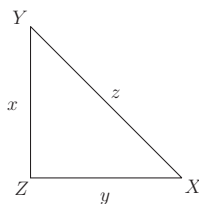
4.3 Direct Proofs

To simplify the terminology used in the following sections, we will refer to the general statements (which require a proof) in the following form: if P , then Q . All four proof techniques will use this form in their definitions.

A direct proof is done by assuming the hypothesis of a statement is true and then working with the given information to reach the conclusion. For example, when considering the direct proof of a statement in the form “if P , then Q ” we would assume that P is true and then use that information to reach our final conclusion of Q . This is referred to as the forward process. In the backward process you pose a more abstract question “How can I conclude the statement Q is true”. See [7]. This then leads to what has to be proven.

This proof technique is the most basic and straightforward method, so to further explain the process, we will show how it works through a few examples.

EXAMPLE 1: Prove, using a direct proof, that if the right triangle, XYZ , shown in the figure below



is isosceles, then the area of the triangle is

$$\frac{z^2}{4}.$$

Proof 1: To prove the statement, we can first assume that $\triangle XYZ$ is an isosceles triangle. Now because $\triangle XYZ$ is a right triangle, we know from the Pythagorean Theorem that $z^2 = x^2 + y^2$. In addition, since $\triangle XYZ$ is isosceles, we know that $x = y$ or $x - y = 0$. We can now square the quantity $(x - y)$, which gives

$$\begin{aligned} 0 &= (x - y) \\ &= (x - y)^2 \\ &= (x - y)(x - y) \\ &= x^2 - 2xy + y^2. \end{aligned}$$

This result can be rewritten as $x^2 + y^2 = 2xy$ and we will substitute back in the equation $z^2 = x^2 + y^2$. We now have $z^2 = 2xy$. The area, A , of any triangle is

$$A = \frac{1}{2}bh$$

where b is the base of the triangle and h is the height of the triangle. From this we know that the area of $\triangle XYZ$ is

$$A = \frac{1}{2}xy.$$

We can now divide both sides of the equation $z^2 = 2xy$ by 4, which leaves us with

$$\frac{z^2}{4} = \frac{1}{2}xy.$$

Hence our final result is that the area of $\triangle XYZ$ is indeed

$$\frac{z^2}{4}.$$

EXAMPLE 2: Prove, using a direct proof, that the product of two odd numbers is odd.

Proof 2: This proposition can be rewritten in the standard form, as

if two numbers are odd, then their product is odd.

We now have a statement from which we can derive a direct proof. We begin by letting m and n both be odd numbers. We will further define them as $m = 2i + 1$ and $n = 2j + 1$ for some integers i and j . When we consider their product, mn , we find that

$$\begin{aligned} mn &= (2i + 1)(2j + 1) \\ &= 4ij + 2i + 2j + 1 \\ &= 2(2ij + i + j) + 1. \end{aligned}$$

Next, let L be the integer that is defined as $L = 2ij + i + j$. From our result

above, we know that $mn = 2L + 1$, that is, twice an integer plus 1. Therefore we have shown that mn is odd.

EXAMPLE 3: Let A , B , C , and D be sets. Prove, using a direct proof, that if $A \subseteq B$ and $C \subseteq D$, then $A \cap C \subseteq B \cap D$.

Proof 3: In the problem we are given that we can assume $A \subseteq B$ and $C \subseteq D$. We let $x \in A \cap C$, which means that $x \in A$ and $x \in C$. Since $x \in A$ and we are given that $A \subseteq B$, then we also know that $x \in B$. Similarly, since $x \in C$ and $C \subseteq D$, we find that $x \in D$. We have now shown that $x \in B$ and $x \in D$, which is equivalent to the statement $x \in B \cap D$. Therefore we have proven that $A \cap C \subseteq B \cap D$.

4.4 Contrapositive Proofs

It is not always possible to prove a statement using a direct argument. When proving certain statements, it is often best to consider the contrapositive. When working with the form “if P , then Q ” the contrapositive would be “if not Q , then not P ” which can also be written as “if $\sim Q$, then $\sim P$ ”. A statement and its contrapositive are equivalent logical statements, so proving either implies the truth of the other. To prove a statement using the contrapositive, begin by supposing that $\sim Q$ is true, and then deduce $\sim P$ is true.

EXAMPLE 1: Prove, by considering the contrapositive statement, that if n^2 is odd, then n is odd.

Proof 1: The first step is to write the contrapositive of the given statement – that is, the statement to consider when starting this proof. The contrapositive is, if n is even, then n^2 is even. So, suppose that n is even. We can write n as $n = 2k$, where k is some integer. Next, consider n^2 , which is found to be

$$\begin{aligned} n^2 &= (2k)^2 \\ &= 2(2k^2). \end{aligned}$$

Since we know that $2(2k^2)$, which is equal to n^2 , is even, then n^2 is even as well. Therefore, we have shown the contrapositive statement and consequently also the original statement, that if n^2 is odd, then n is odd.

EXAMPLE 2: Consider the following definition: a function is 1-1 or injective if for all x and y where $x \neq y$, then $f(x) \neq f(y)$. Now show that when $m \neq 0$ the function $f(x) = mx + b$ is 1-1.

Proof 2: We will first write the contrapositive of our statement we wish to prove, which comes out to be that

$$\text{if } f(x) = f(y), \text{ then } x = y.$$

Now, suppose that $f(x) = f(y)$. From this we can say that $mx + b = my + b$. Further, this means that $mx = my$ where $m \neq 0$. Hence, we can see that $x = y$. We have now shown, using the contrapositive, that if $x \neq y$, then $f(x) \neq f(y)$.

EXAMPLE 3: Let a be a positive integer and let $n > 1$. Prove that if $a^n - 1$ is prime, then $a = 2$.

Proof 3: The first step is to write the contrapositive statement for the original statement. The contrapositive is, if $a \neq 2$, then $a^n - 1$ is not prime. We begin the proof by saying, suppose that $a \neq 2$. Since 1 is a solution to $a^n - 1 = 0$, the quantity $(a - 1)$ is a factor and

$$a^n - 1 = (a - 1)(a^{n-1} + a^{n-2} + a^{n-3} + \cdots + a + 1).$$

Next consider the case when $a = 1$. We have $a^n - 1 = 0$, which is not prime. Now consider the case when $a > 2$. In this case, the factorization is non-trivial, therefore $a^n - 1$ is not prime. Hence, we have proved, using the contrapositive method, our original statement that if $a^n - 1$ is prime, then $a = 2$.

4.5 Proofs by Contradiction

Another technique that comes in handy when trying to prove a statement in the form “if P , then Q ” is a proof by contradiction. This is another way that might be helpful in rearranging the statement to prove. A proof by contradiction begins by supposing P and $\sim Q$ are both true. Using these two pieces of information, the next step is to deduce a contradiction. The difficult part is to determine what the contradiction will be. (Once a contradiction is present, the supposition has been contradicted and the original statement has been proved true.)

EXAMPLE 1: Using a proof by contradiction, show that $\sqrt{2}$ is irrational.

Proof 1: A direct argument approach doesn’t get us anywhere, since assuming $\sqrt{2}$ is irrational does not provide information to work with. To set up a proof by contraposition or contradiction we can write the statement as:

if x is a real number such that $x^2 = 2$, then x is irrational.

Then assuming x is rational, it is still not clear how we can argue that the negation of “ x is a real number such that $x^2 = 2$,” follows. In a proof by contradiction we assume $P \wedge \sim Q$. That is, we assume that x is a real number such that $x^2 = 2$ and that x is a rational number.

So there exists integers p and q such that

$$x = \frac{p}{q}, \quad q \neq 0 \quad \Rightarrow \quad x^2 = \frac{p^2}{q^2}.$$

It still is not clear where the contradiction arises. We further assume p and q

have no common factors. This is the statement that will lead to a contradiction. Since

$$x^2 = \frac{p^2}{q^2} \text{ and } x^2 = 2$$

we have

$$2 = \frac{p^2}{q^2} \Rightarrow p^2 = 2q^2.$$

Then p^2 is even, which implies that p is also even. Write

$$p = 2k, \text{ for some integer } k,$$

so that

$$\begin{aligned} p^2 &= 4k^2 \\ 2q^2 &= 4k^2 \\ q^2 &= 2k^2 \end{aligned}$$

and we also have q^2 and consequently q is even. Since we now know that p and q have a common factor of 2, this contradicts the starting assumption that they have no common factors.

EXAMPLE 2: Give a proof to show that if n is an integer and n^2 is even, then n is even.

Proof 2: Suppose that n^2 is even and n is odd. We will now write n as $n = 2k + 1$ for some $k \in \mathbb{Z}$. From this we find n^2 is calculated as

$$\begin{aligned} n^2 &= (2k + 1)^2 \\ &= (2k + 1)(2k + 1) \\ &= 4k^2 + 4k + 1, \end{aligned}$$

which can be written as $(4k^2 + 4k) + 1$ or $2(2k^2 + 2k) + 1$ and we can easily see that n^2 is odd. Hence, we have a contradiction on n^2 being even and the supposition that n is odd must be false. Therefore, we have proved the original statement that if n^2 is even, then n is even as well.

EXAMPLE 3: Prove, using contradiction, that if a is rational and b is irrational, then $a + b$ is irrational.

Proof 3: To begin, we suppose that a is rational, b is irrational, and $a + b$ is rational. From this we know that

$$a + b = \frac{m}{n}$$

and

$$a = \frac{c}{d},$$

so when we consider $(a + b) - a$, we find that

$$\begin{aligned} (a + b) - a &= \frac{m}{n} - \frac{c}{d} \\ &= \frac{dm - cn}{dn} \end{aligned}$$

which is a rational quantity. We have just shown that $(a + b) - a$ is rational, and since $(a + b) - a = b$, then we have shown that b is rational. Now we have a contradiction on b , therefore we have proved the original statement that if a is rational and b is irrational, then $a + b$ is irrational.

4.6 Mathematical Induction

Mathematical induction is a technique of proof that is used to prove a statement, depending on a parameter n , is true for all natural numbers $n = 1, 2, 3, \dots$. Proofs requiring this method involve some property $P(n)$ that is defined for all positive integers. Using mathematical induction, we must show that $P(n)$ is true for all positive integers, that is, the truth set for $P(n)$ is all positive integers. There are two steps to this process: the *base step* and the *inductive step*.

Begin with the base step by showing that the statement is true for the base case, $n = 1$, or showing that $P(1)$ is true. The base case can start at any natural number k , but then the proof is that $P(n)$ is true for all $n \geq k$. Once you have acquired a base, it is time to move on to the inductive step.

In the inductive part of the proof, **suppose** that $P(n)$ is true for some index n , and then show the statement $P(n + 1)$ is true.

Here is a summary of the steps in a proof by induction,

1. base step: show $P(1)$ holds
2. inductive step: suppose $P(n)$ holds
3. goal: using the fact that $P(n)$ holds, conclude $P(n + 1)$ holds as well.

This process is best explained through example. The following three examples portray classic proofs that require the use of mathematical induction.

EXAMPLE 1: Prove, using mathematical induction, that for all $n \geq 1$,

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}.$$

Proof 1: We will begin with the base step and consider $n = 1$, so the left hand

side of the summation formula is

$$\sum_{i=1}^1 i = 1$$

and the right hand side is

$$\frac{1(1+1)}{2} = 1.$$

Consequently, the formula holds for the case when $n = 1$. Next we will start the inductive hypothesis and suppose that $P(n)$ or

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

holds. We need to show the statement $P(n+1)$ holds, that is, we need to show

$$\sum_{i=1}^{n+1} i = \frac{(n+1)((n+1)+1)}{2} = \frac{(n+1)(n+2)}{2}.$$

By this inductive hypothesis,

$$\begin{aligned} \sum_{i=1}^{n+1} i &= \sum_{i=1}^n i + (n+1) \\ &= \frac{n(n+1)}{2} + (n+1) \\ &= \frac{n(n+1)}{2} + \frac{2(n+1)}{2} \\ &= \frac{(n+1)(n+2)}{2}. \end{aligned}$$

Therefore, we have proved, using mathematical induction, that for all $n \geq 1$,

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}.$$

EXAMPLE 2: Prove that for all $n \geq 1$,

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}.$$

Proof 2: We will begin with the base step and consider $n = 1$, so the left hand side of the summation formula is

$$\sum_{i=1}^1 i^2 = 1$$

and the right hand side is

$$\frac{1(1+1)(2(1)+1)}{6} = 1.$$

Consequently, the formula holds for the case when $n = 1$. Next we will start the inductive hypothesis and suppose that $P(n)$ or

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

holds. We need to show the statement $P(n+1)$ holds, that is, we need to show

$$\sum_{i=1}^{n+1} i^2 = \frac{(n+1)((n+1)+1)(2(n+1)+1)}{6} = \frac{(n+1)(n+2)(2n+3)}{6}.$$

By this inductive hypothesis,

$$\begin{aligned} \sum_{i=1}^{n+1} i^2 &= \sum_{i=1}^n i^2 + (n+1)^2 \\ &= \frac{n(n+1)(2n+1)}{6} + (n+1)^2 \\ &= \frac{n(n+1)(2n+1)}{6} + \frac{6(n+1)^2}{6} \\ &= \frac{1}{6}(n+1)[n(2n+1) + 6(n+1)] \\ &= \frac{1}{6}(n+1)[2n^2 + 7n + 6] \\ &= \frac{1}{6}(n+1)(n+2)(2n+3). \end{aligned}$$

Therefore, we have proved, using mathematical induction, that for all $n \geq 1$,

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}.$$

EXAMPLE 3: Prove, by mathematical induction, that $3^n - 1$ is divisible by 2 for all $n \geq 1$.

Proof 3: We will begin with the base step and consider $n = 1$, so

$$3^1 - 1 = 2$$

which is obviously divisible by 2. Next, we will suppose that $P(n)$ holds. We need to show that $P(n+1)$ holds, or in other words we must show that $3^{n+1} - 1$

is divisible by 2 as well. We find that

$$\begin{aligned}3^{n+1} - 1 &= 3 \cdot 3^n - 1 \\ &= 3 \cdot 3^n - 3 + 2 \\ &= 3(3^n - 1) + 2.\end{aligned}$$

Since we have assumed that $3^n - 1$ is divisible by 2, and 2 is divisible by 2, we have $3^{n+1} - 1$ is divisible by 2. Therefore, the proof is complete and we have successfully shown that $3^n - 1$ is divisible by 2 for all $n \geq 1$.

Chapter 5

Using Graphics

5.1 Introduction

As many say, a picture is worth a thousand words. It is often important in math to clarify or strengthen an argument by using a figure or a graph. Such graphics can be created and inserted into a \LaTeX document with very little effort. This chapter will walk through the process from creating the graph to actually inserting it in a document and including appropriate labels.

5.2 Creating Graphs Using Maple

5.2.1 Maple Basics

To create a graph using Maple, start by opening a blank document. The first command that is necessary should be written exactly as follows, “with(plots):” or “with(plots);”. Depending on whether or not you would like to view the commands that are part of the plots package in Maple, any command in Maple is ended with a semicolon. The semi-colon shows the contents of the package and the colon hides the contents. Ending a command with a colon suppresses the default output. After you have typed in the command, hit Enter. You will see a new execution line (\gt) appear where the next command may be entered.

The command “plot()” is used to graph a function. Inside the parentheses is where you specify the function or equation to plot and one or more plot options that describe the graph. The basic form of the plot command is

```
plot(expression specifying what to plot, plot options separated by commas);
```

As with the syntax in any programming language, there is a good deal of flexibility for the actual form of the plot command.

The standard symbols are used for the arithmetic operations, which are also needed when defining, for example, functions. A carrot (\wedge) is used for an exponent, an asterisk ($*$) is used for multiplication, a forward slash ($/$) is

used for division, and parentheses are always a good idea to clarify which terms go together.

In Maple there is a long list of plot options you can use. We mention only a few here that will allow you to plot most basic graphs. Most basic functions will look fine if you have the x -axis go from -5 to 5 and the y -axis go from -5 to 5 . Depending on the function, you can make those limits anything you like. The bounds on x and y are specified in the form (separated by a comma) “ $x = -5..5, y = -5..5$ ” with two periods in between the limits.

Another aspect of the figure that you may want to change is the color of your graph. Maple will, by default, draw the graph in red (or multiple colors whenever more than one graph is displayed) unless you specify otherwise. The option “color=black” will ensure that everything is graphed in black. This is important because when you print your \LaTeX document (using a black and white printer), the figure will not look light or fuzzy as if it were in another color.

In preparation for converting this image into an Adobe Illustrator .eps file, we want to remove the tick marks on the axes of the graph, which is done by including the options “xtickmarks=0” and “ytickmarks=0”. We are going to place the curve we are creating in Maple on a standardized set of axes, so the automated tick marks are unnecessary here.

Finally, to be sure that our curve is as smooth as possible, we want to increase the number of points that we want Maple to include when it draws the graph. This is done using the option called “numpoints”. For example, “numpoints=1000”. The number can be changed to as many or few points as you desire, but 1000 guarantees us a smooth and solid picture.

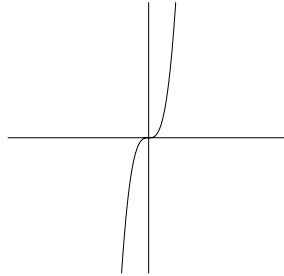
When plots use the same scales on the x -axis and y -axis it is a good idea to include the option “scaling=constrained”, which will guarantee the scaling on both axes is identical.

5.2.2 An Example

Considering everything explained about how to create a graph of a simple function, we will go through a quick example. For instance, say we want to graph the function $y = f(x) = 5x^3$. The command in the Maple file look like this:

```
> plot((5*x^3),x=-5..5,y=-5..5,scaling=constrained,color=black,  
xtickmarks=0,ytickmarks=0,numpoints=1000);
```

The output is the graph shown in the figure.



As you can see, the function has been graphed, but it is neither clearly labeled nor aesthetically appealing. The next step is to import the graph into Adobe Illustrator and give it a more professional appearance.

5.3 Converting Maple Files

5.3.1 Exporting Graphs to Adobe Illustrator

The `plot` command in Maple, by default sends the output to the monitor screen, placed in the Maple document immediately following the command. The location of the output can be sent to a file as well. You can name this file anything you like and you can determine exactly where it will be saved on your computer. This command must be executed before plotting the function, so insert an execution line above the `plot` command so that you have room to include this line. The form of the command is:

```
> plotsetup(ps,plotoutput='C:\\Destination Folder Name\\Filename.eps',  
plotoptions='portrait,noborder,xaxis=1.5,yaxis=1.5');
```

There are two things to which you should pay attention. As you can see, you must tell Maple where to save this `.eps` file using “`C:\\ Destination Folder Name`”. You must also tell Maple what you want the file to be called by specifying “`Filename.eps`”. Your file should be named with a single word and you must include the `.eps` for encapsulated postscript files. The purpose of the different fields in the command are described in the table.

Field	Description
ps	postscript file
plotoutput=''	The destination of the file is specified inside the single quotes including any folder names.
plotoptions=''	Specifies the characteristic of the plot <ul style="list-style-type: none"> • portrait: vertical orientation • noborder: plot is not enclosed in a rectangle • xaxis=1.5,yaxis=1.5: the length of the axes in inches

When the `plotseupt` command followed by the `plot` command are executed, the graph will not appear on the screen, but when you open the destination folder, you should see an `.eps` file with the name you specified. This file is now ready to be altered and improved using Adobe Illustrator.

5.3.2 Adapting the Appearance

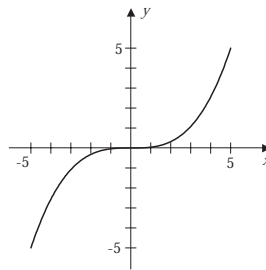
Once you are in the Adobe Illustrator program, open the `.eps` file you created using Maple. There are a few windows in Adobe Illustrator with which you will want to be familiar. (All of these windows and many others can be activated by selecting them from the “Window” menu.) The “Layers” window is handy because it allows you to create pictures as separate layers so later you can look at part of the figure while another part is hidden. Also, the “Transform” window is important in resizing the curve to fit appropriately on the new set of axes. Finally, the “Tools” window contains the selection tools that you will need to select the curve independently from the axes. Once you are comfortable with the appearance of Adobe Illustrator and its many components, you are ready to convert your image.

This process is done using a nice, standardized set of axes previously created by Dr. Jim DeFranza. Once you have your graph open, next you can open the `.eps` file that contains the new set of axes. Make sure you immediately save the axes under another file name using “Save As...” from the File menu. You don’t want to forget to do this and risk losing the file of the good axes. (I would recommend having multiple copies of the good axes file in various places on your computer, just in case!)

Using the “Direct Selection Tool” from the “Tools” window, click on the curve in your Maple `.eps` file, copy it, go to the new axes file, and paste it in the graphic layer. The image will be enormous and not in the right place on the axes, but it is an easy thing to fix. Simply go to the “Transform” window and in the box labeled “X:” put 0, in the box labeled “Y:” put 0, in the box labeled “W:” put 100, and in the box labeled “H:” put 100. This will center your curve in the correct place on the new axes and resize it appropriately. Save this new

and improved version of your graph, and you are ready to insert it into your L^AT_EX document.

Continuing our example from the previous section, the new and revised .eps file would now appear as shown in the figure.



5.4 Including Graphics in L^AT_EX

Once you have created and modified your .eps file, it is a single-line command in L^AT_EX to include the figure in a document. It usually looks best if you center the figure, which can be done using either using the center environment or the centerline command. The L^AT_EX commands that place your figure in your document are as follows:

```
\begin{center}
\resizebox{150pt}{!}{\includegraphics{Filename.eps}}
\end{center}
```

You specify the name of your file and how big you want it to appear. The “resizebox” command allows you to change the width and height of a figure. The ! in the second field makes the height the same as the width. It is important to save your graphics file in the same folder as your .tex file because otherwise L^AT_EX will have difficulty finding the graphics file. It can be done in other ways, but this is the easiest. The only other basic alteration you may want to perform is placing a label on the curve.

5.5 Placing Labels on Graphics

Now we can think about putting a label on your figure. If you want to include a label like $y = f(x)$ next to the curve, we have to open the .eps file in Adobe Illustrator again.

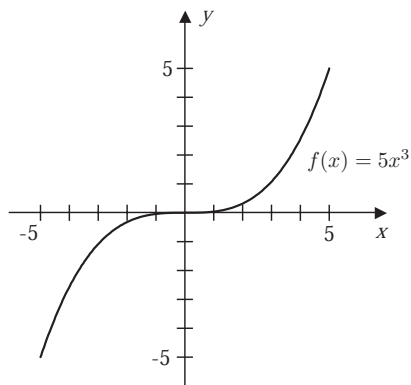
We could type the label for the graph in Adobe Illustrator, but if it is complex at all, it will look awful and be hard to do. The solution can be found using the psfrag package. All you have to do is place a single letter, “A” for example, where you want the label in your figure. Save the file and go back to your L^AT_EX document.

It is yet another short command

```
\psfrag{A}{\scalebox{0.75}{function}}
```

with your function defined (as you want it to appear in the label) in the second set of curly braces. This command should be placed in the line above the command where you include your .eps file. The psfrag package does not usually work with the basic viewing program, so the output of your L^AT_EX document might be incorrect. To fix this problem, just hit the L^AT_EX button about three or four times, then hit the dvi→ps button, and finally click on the ps→pdf button and your document will be opened as a .pdf file in which your figure should appear in the correct form.

Our original example gives the following figure.



The final commands compiled (the example figure was named Examp3.eps) to make the figure above are as follows:

```
\psfrag{A}{\scalebox{0.75}{f(x) = 5x^3}}
\begin{center}
\resizebox{2in}{!}{\includegraphics{Examp3.eps}}
\end{center}
```

Depending on your specific MiK_TE_X installation you may have to load the psfrag package. If necessary, in the preamble include the following statement.

```
\usepackage{psfrag}
```

Chapter 6

Project Analysis

6.1 Why Write This Document?

My inspiration to write this document sparked from my experiences as a student, a mathematics major and applied statistics minor, here at St. Lawrence University, along with my future career plans to become a teacher. I have had been fortunate to have taken five courses with my advisor, Jim DeFranza, who requires that his students type up their homework. The problems that arise initially lie in the students, but as I have found, it can easily be fixed. Most students have never had the experience of writing up homework solutions for a math class, using explanatory sentences and all. They have only been required to write solutions up neatly by hand, so Dr. DeFranza is often met with confusion and frustration when his students begin to type up the homework problem sets in his classes.

One of the courses that I took with Dr. DeFranza was Mathematics 248: Technical Writing, which consisted of myself and six other students. It allowed us to work in a small and intimate seminar environment, and our group focused on how to use \LaTeX to create documents along with how to recognize the characteristics of good and bad writing. We spent the semester actively writing up various kinds of documents to practice our \LaTeX typing skills. We also edited the writing of our peers which instilled in us the theory that writing is a process. I now realize how beneficial that course was and that is another reason that I chose to write this document. I believe that students would benefit from a mandatory course in which perspective mathematics majors, and even applied statistics majors, learned how to use \LaTeX and how to write documents that have a professional appearance. This document is an example of some of the topics that could be taught in such a course. I must say that it is from Dr. DeFranza's practices and teaching that I have learned the importance of writing in mathematics. I can now look at a stack of homework assignments written by students in Mathematics 135: Calculus 1 and immediately critique the writing. The most important and valuable experience about writing up and

typing up solutions is being able to articulate thoughts into words on a page, clearly explaining ideas and the information in the problem and the solution. This is a skill that all majors in the math and sciences should master before the time they graduate.

6.2 L^AT_EX Workshop

During the present spring semester at St. Lawrence University, I was employed as the teaching assistant (TA) for Dr. DeFranza's course on complex analysis. This was the second year that he required students not only to type up their homework, but to do so using L^AT_EX instead of the program Scientific Notebook. The previous year, when he initially attempted using L^AT_EX as the primary means of typing up homework, I was a student in the class and the TA for help with the use of the program. This year went much better and I encountered fewer students in need of assistance, and the difference was caused by one single event. During the first week of classes, I held a workshop, for the students in the class, in which I taught them the basics of the program L^AT_EX and walked them through a few examples. Dr. DeFranza and I agreed that a workshop would benefit all, but I never anticipated it would be as helpful as they found it to be. The only help I gave out as a TA this spring was about content and minor L^AT_EX questions or problems. I now understand that a workshop on how to get started, like this one which only lasted an hour or so, is extremely beneficial to students who want or need to learn the program.

6.3 L^AT_EX vs. Scientific Notebook

In my mind, Scientific Notebook is the easy and lazy way to use L^AT_EX. It is a wonderful program to use in the introductory courses, but by the time students choose to pursue a major in mathematics, they should become familiar with L^AT_EX as well. L^AT_EX is the action behind the scenes of the Scientific Notebook program. The .tex file created in Scientific Notebook can also be opened in L^AT_EX and slightly altered in order for it to work. The significant difference between these two programs is the ease of creating a personalized document and the beauty of the appearance. I must say that the output generated from L^AT_EX compared to the same output generated from Scientific Notebook will be phenomenally different. I initially learned how to type in Scientific Notebook, and that was fine until I took the Technical Writing course with Dr. DeFranza and I now refuse to use anything but L^AT_EX. I feel that L^AT_EX makes all documents look more professional and are of an overall higher quality. I believe that all students, especially perspective mathematics or statistics majors, should have the ability to use L^AT_EX.

6.4 Application to Other Courses

The most fascinating part of the program, \LaTeX , is that it can be used to write documents for any other area of study. Ever since the course I took with Dr. DeFranza, Mathematics 247: Technical Writing, I have used \LaTeX to write up my homework and other assignments in all my other math courses and even courses like economics. To illustrate how \LaTeX can be used in various contexts, other than for writing proofs in Bridge to Higher Mathematics, I would like to share some of my work. It is amazing to me to look back at the homework I handed in my first year in college and compare them with homework from my junior year or even this document. The change is phenomenal and I am proof that with practice, determination, and the desire to succeed, anything can be done. I hope that you have enjoyed this document and I will conclude with my excerpts of work from other courses.

Mathematics 217: Linear Algebra

EXAMPLE: Determine if the set

$$\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} -7 \\ 5 \\ 4 \end{bmatrix}$$

is a base for \mathbb{R}^3 .

Solution: To determine whether or not the given set is a basis, we must check to see if it is linearly independent and then see if the span of the set is equal to \mathbb{R}^3 . We will start by checking for linear independence. The first step is to set a linear combination of the vectors equal to the zero vector, that is,

$$\alpha_1 \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} + \alpha_2 \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} + \alpha_3 \begin{bmatrix} -7 \\ 5 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

which yields the system of equations,

$$\begin{aligned} 2\alpha_1 + \alpha_2 - 7\alpha_3 &= 0 \\ -2\alpha_1 - 3\alpha_2 + 5\alpha_3 &= 0 \\ \alpha_1 + 2\alpha_2 + 4\alpha_3 &= 0. \end{aligned}$$

We will now put the system of equations into matrix form and solve for $\alpha_1, \alpha_2,$

and α_3 . The system becomes

$$\begin{aligned} & \begin{bmatrix} 2 & 1 & -7 & 0 \\ -2 & -3 & 5 & 0 \\ 1 & 2 & 4 & 0 \end{bmatrix} : R_1 \leftrightarrow R_3 \\ \Rightarrow & \begin{bmatrix} 1 & 2 & 4 & 0 \\ -2 & -3 & 5 & 0 \\ 2 & 1 & -7 & 0 \end{bmatrix} : 2R_1 + R_2 \rightarrow R_2; -2R_1 + R_3 \rightarrow R_3 \\ \Rightarrow & \begin{bmatrix} 1 & 2 & 4 & 0 \\ 0 & 1 & 13 & 0 \\ 0 & -3 & -15 & 0 \end{bmatrix} : 3R_2 + R_3 \rightarrow R_3 \\ \Rightarrow & \begin{bmatrix} 1 & 2 & 4 & 0 \\ 0 & 1 & 13 & 0 \\ 0 & 0 & 24 & 0 \end{bmatrix}. \end{aligned}$$

The reduced matrix is equivalent to the system of equations,

$$\alpha_1 + 2\alpha_2 + 4\alpha_3 = 0$$

$$\alpha_2 + 13\alpha_3 = 0$$

$$24\alpha_3 = 0$$

from which we can simply deduce that $24\alpha_3 = 0$ implies that $\alpha_3 = 0$. We then have $\alpha_2 + 13\alpha_3 = 0$, so $\alpha_2 = 0$ as well. Finally, we see that $\alpha_1 + 2\alpha_2 + 4\alpha_3 = 0$ and we have $\alpha_1 = 0$. This shows that the set is indeed linearly independent.

The next step is to check whether or not the span of the set is equal to \mathbb{R}^3 . To do this, we set up the same system of equations, but instead of setting it equal to the zero vector, we will now set it equal to a vector with arbitrary vector components a, b , and c . If α_1, α_2 , and α_3 can be written in terms of a, b , and c , then the span of the set is equal to \mathbb{R}^3 . When we do this, the linear combination becomes

$$\alpha_1 \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} + \alpha_2 \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} + \alpha_3 \begin{bmatrix} -7 \\ 5 \\ 4 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

yielding the system of equations

$$2\alpha_1 + \alpha_2 - 7\alpha_3 = a$$

$$-2\alpha_1 - 3\alpha_2 + 5\alpha_3 = b$$

$$\alpha_1 + 2\alpha_2 + 4\alpha_3 = c.$$

It is now ready to be transformed to matrix form, and we will attempt to solve the system. The matrix reduction is shown below.

$$\begin{aligned} & \begin{bmatrix} 2 & 1 & -7 & a \\ -2 & -3 & 5 & b \\ 1 & 2 & 4 & c \end{bmatrix} : R_1 \leftrightarrow R_3 \\ \Rightarrow & \begin{bmatrix} 1 & 2 & 4 & c \\ -2 & -3 & 5 & b \\ 2 & 1 & -7 & a \end{bmatrix} : 2R_1 + R_2 \rightarrow R_2; -2R_1 + R_3 \rightarrow R_3 \\ \Rightarrow & \begin{bmatrix} 1 & 2 & 4 & c \\ 0 & 1 & 13 & 2c + b \\ 0 & -3 & -15 & -2c + a \end{bmatrix} : 3R_2 + R_3 \rightarrow R_3 \\ \Rightarrow & \begin{bmatrix} 1 & 2 & 4 & c \\ 0 & 1 & 13 & 2c + b \\ 0 & 0 & 24 & 4c + 3b + a \end{bmatrix}. \end{aligned}$$

The reduced matrix is equivalent to the system

$$\begin{aligned} \alpha_1 + 2\alpha_2 + 4\alpha_3 &= c \\ \alpha_2 + 13\alpha_3 &= 2c + b \\ 24\alpha_3 &= 4c + 3b + a \end{aligned}$$

and we must now solve for the three variables. In doing this, we find α_3 by

$$\begin{aligned} 24\alpha_3 &= 4c + 3b + a \\ \alpha_3 &= \frac{4c + 3b + a}{24} \\ \alpha_3 &= \frac{1}{6}c + \frac{1}{8}b + \frac{1}{24}a. \end{aligned}$$

Using α_3 we can calculate α_2 from

$$\begin{aligned}\alpha_2 + 13\alpha_3 &= 2c + b \\ \alpha_2 &= 2c + b - 13\alpha_3 \\ \alpha_2 &= 2c + b - 13\left(\frac{1}{6}c + \frac{1}{8}b + \frac{1}{24}a\right) \\ \alpha_2 &= 2c + b - 13\frac{1}{6}c - \frac{13}{8}b - \frac{13}{24}a \\ \alpha_2 &= -\frac{1}{6}c - \frac{5}{8}b - \frac{13}{24}a\end{aligned}$$

and finally using α_3 and α_2 , we have

$$\begin{aligned}\alpha_1 + 2\alpha_2 + 4\alpha_3 &= c \\ \alpha_1 &= c - 2\alpha_2 - 4\alpha_3 \\ \alpha_1 &= c - 2\left(-\frac{1}{6}c - \frac{5}{8}b - \frac{13}{24}a\right) - 4\left(\frac{1}{6}c + \frac{1}{8}b + \frac{1}{24}a\right) \\ \alpha_1 &= c + \frac{1}{3}c + \frac{5}{4}b + \frac{13}{12}a - \frac{2}{3}c - \frac{1}{2}b - \frac{1}{6}a \\ \alpha_1 &= \frac{2}{3}c + \frac{3}{4}b + \frac{11}{12}a.\end{aligned}$$

Therefore, the span of the given set is \mathbb{R}^3 . Since we have already shown linear independence, we are finished and we have shown that this set is a base for \mathbb{R}^3 .

Mathematics 306: Complex Analysis

EXAMPLE: In parts (a)-(d) evaluate the integral

$$\int_C \frac{z^4}{z^2 + 4} dz$$

for the given contour C .

- a. $|z - 2i| = 1$
- b. $|z + 2i| = 1$
- c. $|z| = 4$
- d. $|z - 2| = 2$

Solution: We can first factor the denominator of the integral, giving us

$$\int_C \frac{z^4}{z^2 + 4} dz = \int_C \frac{z^4}{(z + 2i)(z - 2i)} dz.$$

- a. For this part of the problem, we are considering the contour $|z - 2i| = 1$, which is a circle centered at the point $(0, 2)$ with a radius of length 1. When we

consider the two points at which our function is not analytic, we can see that we are only concerned with $z = 2i$, where the function is not analytic. Once we recognize this, we can rewrite our integral as

$$\int_{|z-2i|=1} \frac{z^4}{z^2+4} dz = \int_{|z-2i|=1} \frac{\frac{z^4}{(z+2i)}}{(z-2i)} dz$$

Since the function in the numerator is analytic in and on the contour, we can now use Cauchy's Integral Formula to calculate this integral. It is stated as

$$f^{(n)}(z_0) = \frac{n!}{2\pi i} \int_C \frac{f(z)}{(z-z_0)^{n+1}} dz.$$

When we look at our problem, we can see that our function is

$$f(z) = \frac{z^4}{(z+2i)},$$

$n = 0$, and $z_0 = 2i$. We can then plug z_0 in for $f(z)$ which gives

$$\begin{aligned} f(z) &= \frac{z^4}{(z+2i)} \\ f(2i) &= \frac{(2i)^4}{((2i)+2i)} \\ &= \frac{16}{4i}. \end{aligned}$$

We now have everything we need to manipulate Cauchy's Integral Formula and compute our integral, which gives us

$$\begin{aligned} f(2i) &= \frac{0!}{2\pi i} \int_{|z-2i|=1} \frac{\frac{z^4}{(z+2i)}}{(z-2i)} dz \\ 8\pi &= \int_{|z-2i|=1} \frac{\frac{z^4}{(z+2i)}}{(z-2i)} dz. \end{aligned}$$

Therefore, the original integral is equal to 8π .

b. For this part of the problem, we are considering the contour $|z+2i|=1$, which is a circle centered at the point $(0, -2)$ with a radius of length 1. When we consider the two points at which our function is not analytic, we can see that we are only concerned with $z = -2i$. Once we recognize this, we can rewrite our integral as

$$\int_{|z+2i|=1} \frac{z^4}{z^2+4} dz = \int_{|z+2i|=1} \frac{\frac{z^4}{(z-2i)}}{(z+2i)} dz$$

We can now use Cauchy's Integral Formula to calculate this integral. Where

our function is

$$f(z) = \frac{z^4}{(z - 2i)},$$

$n = 0$, and $z_0 = -2i$. We can then plug z_0 in for $f(z)$ which gives

$$\begin{aligned} f(z) &= \frac{z^4}{(z - 2i)} \\ f(-2i) &= \frac{(-2i)^4}{((-2i) - 2i)} \\ &= -\frac{16}{4i}. \end{aligned}$$

We now have everything we need to manipulate Cauchy's Integral Formula and compute our integral which gives us the following for our final integral

$$\begin{aligned} f(-2i) &= \frac{0!}{2\pi i} \int_{|z+2i|=1} \frac{\frac{z^4}{(z-2i)}}{(z+2i)} dz \\ -8\pi &= \int_{|z+2i|=1} \frac{\frac{z^4}{(z-2i)}}{(z+2i)} dz. \end{aligned}$$

Therefore, the original integral is equal to -8π .

c. For this part we are considering the contour $|z| = 4$, which is a circle centered at the origin with a radius of length 4. When we look at where the two points, at which the function is not analytic, lie, we can see that they are both inside of the given circle. This creates a problem because there are now two places at which this function is not analytic in the given domain. This means that we must break up the integral by using partial fractions. The term z^4 can be ignored in the partial fraction decomposition. When we do this, we find that

$$\begin{aligned} \frac{1}{(z+2i)(z-2i)} &= \frac{A}{z+2i} + \frac{B}{z-2i} \\ 1 &= A(z-2i) + B(z+2i) \\ 1 &= (A+B)z + (B-A)2i \end{aligned}$$

Therefore we must have that $B + A = 0$ and $B - A = -\frac{i}{2}$, which means that $2B = -\frac{i}{2}$, so we find $B = -\frac{i}{4}$ and $A = \frac{i}{4}$. We can now rewrite our integral as follows

$$\int_{|z|=4} \frac{z^4}{(z+2i)(z-2i)} dz = \frac{1}{4} \int_{|z|=4} \frac{iz^4}{z+2i} - \frac{1}{4} \int_{|z|=4} \frac{iz^4}{z-2i}.$$

We must now use Cauchy's Integral Formula to calculate these two integrals. For both integrals, $f(z) = iz^4$, but for the first integral, $z_0 = -2i$ and for the second integral, $z_0 = 2i$. We can see that both singular points are of order 1, so

the combination of these two integrals yields

$$\frac{1}{4} \int_{|z|=4} \frac{iz^4}{z+2i} - \frac{1}{4} \int_{|z|=4} \frac{iz^4}{z-2i} = \frac{2\pi i}{4}(16i - 16i) = 0.$$

Therefore, the original integral comes out to be 0.

d. For this part we are considering the contour $|z - 2| = 2$ which is the circle centered at the point $(2, 0)$ with a radius of length 2. It is easy to see that neither of our critical points fall in or on this circle, therefore the given function is analytic everywhere in and on the domain, which means that by Cauchy's Theorem,

$$\int_{|z-2|=2} \frac{z^4}{z^2+4} dz = 0.$$

Mathematics 316: Ring Theory:

EXAMPLE

[a] Define each of the following terms and give some examples:

[1] The invertible elements or units $U(A)$ of a ring A with identity 1_A .

[2] $F : X \rightarrow Y$ is a surjective map.

[3] $F : A \rightarrow B$ is a homomorphism of rings A, B with identities 1_A and 1_B respectively.

[b] Let U be a universe with just the elements $\heartsuit, \spadesuit, \diamondsuit, \clubsuit$ and V the universe with just the elements a, b, c . Determine the number of surjective maps, $F : U \rightarrow V$.

Solution:

[a]

[1] Let A be a commutative ring. An element $\alpha \in A$ is said to be invertible or called a unit if and only if there exists an element $\beta \in A$ such that $\alpha \otimes \beta = 1$ and $\beta \otimes \alpha = 1$.

Example: Let $A = \mathbb{Z}_7$, then the element 3 is a unit because $3 \otimes 5 = 1 = 5 \otimes 3$, so $3^{-1} = 5$ and $5^{-1} = 3$.

[2] A function $F : X \rightarrow Y$ is said to be a surjective map provided that every element of Y is the image under F of at least one element of X . This can also be written as: if $y \in Y$, then there exists $x \in X$ such that $F(x) = y$.

Example: If $X = \{\alpha, \beta, \delta\}$, $Y = \{m, n, k\}$, and $F : X \rightarrow Y$, then the map

$$F(\alpha) = m$$

$$F(\beta) = n$$

$$F(\delta) = k$$

is a surjective map.

[3] Let A and B be rings. A function $F : A \rightarrow B$ is a homomorphism if the following properties hold

$$F(a + b) = F(a) + F(b)$$

and

$$F(ab) = F(a)F(b)$$

for all $a, b \in A$. The final statement that must hold is that

$$F(1_A) = 1_B$$

or in words, the identity of A should be mapped to the identity of B .

Example: $F : A \rightarrow B$ when $A = \mathbb{Z}$ and $B = \mathbb{Z}_7$ which can also be written as $F : \mathbb{Z} \rightarrow \mathbb{Z}_7$.

[b] For this problem, we must first figure out how many maps there are, $F : U \rightarrow V$. Since we can map the four elements in U three ways to each of the elements in V , there are $3 \times 3 \times 3 \times 3 = 3^4$ or 81 possible maps. Not all of these maps are surjective though. To figure out how many surjective maps there are, we will construct a table for each element in V showing the different surjective combinations there are for the four elements in U . The table of combinations for a , where b or c would alternate in the spaces left, is shown in the Table for (a).

Table for a

$f(\heartsuit) =$	a	a	a	b/c	b/c	b/c
$f(\spadesuit) =$	a	b/c	b/c	a	a	c/b
$f(\diamondsuit) =$	b/c	a	c/b	a	c/b	a
$f(\clubsuit) =$	c/b	c/b	a	c/b	a	a
Surjective Maps	2	2	2	2	2	2

We can see from the table that there are 2 surjective maps for each of the six combinations, which means there are 12 surjective maps when a is fixed and b and c alternate. Similarly we can construct a table of combinations for b , where a or c would alternate in the spaces left. See the Table for (b).

Table for b

$f(\heartsuit) =$	b	b	b	a/c	a/c	a/c
$f(\spadesuit) =$	b	a/c	a/c	b	b	c/a
$f(\diamondsuit) =$	a/c	b	c/a	b	c/a	b
$f(\clubsuit) =$	c/a	c/a	b	c/a	b	b
Surjective Maps	2	2	2	2	2	2

Again we can see that there are 12 more surjective maps here. Finally, the table of combinations for c , where a or b would alternate in the spaces left is shown in the Table for (c).

Table for c

$f(\heartsuit) =$	c	c	c	b/a	b/a	b/a
$f(\spadesuit) =$	c	b/a	b/a	c	c	a/b
$f(\diamondsuit) =$	b/a	c	a/b	c	a/b	c
$f(\clubsuit) =$	a/b	a/b	c	a/b	c	c
Surjective Maps	2	2	2	2	2	2

We have 12 more surjective maps this time. Therefore, altogether we have $12 + 12 + 12$ or 36 surjective maps out of the 81 total maps.

Mathematics 325: Probability

EXAMPLE:

Let X measure the price (in dollars) of a commodity that might fluctuate between \$ 0.30 and \$ 0.40 per unit. Let Y measure the sales of the commodity (in 10,000's of units). We will assume that the joint density for (X, Y) is given by

$$f(x, y) = \begin{cases} 5xe^{-xy}, & \text{if } 0.20 < x < 0.40 \text{ and } 0 < y \\ 0, & \text{otherwise.} \end{cases}$$

- What is the probability that the price will be less than \$ 0.30 and the sales will exceed 20,000?
- Find the marginal densities for both price and sales.
- Find the average price and average sales.
- Of course, the revenue (R) is just the product of the price times sales. Find the expected revenue.
- Find the probability that the sales will be less than 30,000 units if the price is set at \$ 0.25.
- Calculate the correlation between price and sales. Give a reasonable interpretation for the sign (positive or negative) of the correlation in this particular situation.

Solution:

- The probability that the price will be less than \$ 0.30 and the sales will

exceed 20,000 is calculated by

$$P(X < 0.30, Y > 20,000) = \int_2^{\infty} \int_{.2}^{.3} 5xe^{-xy} dx dy = .303771$$

(b) The marginal densities are calculated as

$$f_x(x) = \int_0^{\infty} 5xe^{-xy} dy = \lim_{y \rightarrow \infty} -5e^{-xy} + 5 \quad \text{if } .2 < x < .4$$

$$f_y(y) = \int_{.2}^{.4} 5xe^{-xy} dx = \frac{-.5e^{-.4y} - 2ye^{-.4y} + .5e^{(-.2y)} + ye^{-.2y}}{y^2} \quad \text{if } 0 < y$$

(c) The average price and average sales are calculated as

$$E(X) = \int_0^{\infty} \int_{.2}^{.4} 5x^2 e^{-xy} dx dy = .3$$

$$E(Y) = \int_0^{\infty} \int_{.2}^{.4} 5xy e^{-xy} dx dy = 3.465736$$

(d) Since $R = XY$, the expected revenue is calculated as

$$E(R) = E(XY) = \int_0^{\infty} \int_{.2}^{.4} 5x^2 y e^{-xy} dx dy = 1$$

(e) We are trying to find $P(Y < 30000 | X = .25)$, so the probability that the sales will be less than 30,000 units when the price is set at \$ 0.25, is calculated as

$$f(y|x) = \frac{f(x,y)}{f_x(x)} = \frac{5xe^{-xy}}{\lim_{y \rightarrow \infty} -5e^{-xy} + 5}$$

When $x = .25$, we have

$$f(y|x = .25) = .25e^{-.25y}$$

$$P(Y < 30000 | X = .25) = \int_{.25}^3 .25e^{-.25y} dy = .46704$$

(f) To calculate the correlation between price and sales, we must first find the following things: $E(X)$, $E(Y)$, $E(XY)$, $E(X^2)$, $V(X)$, σ_X , $E(Y^2)$, $V(Y)$, and σ_Y . In part (c) we found that $E(X) = .3$ and $E(Y) = 3.465736$, and from part (d) we calculated $E(XY) = 1$. To calculate the correlation, we must first calculate the covariance, which we can do from the information we have already

found. The covariance is given by

$$\text{Cov}(X, Y) = E(XY) - \mu_X \mu_Y = 1 - (.3)(3.465736) = 1 - 1.03972 = -0.03972$$

We can now finish the remaining calculations. They are as follows:

$$\begin{aligned} E(X^2) &= \int_0^\infty \int_{.2}^{.4} 5x^3 e^{-xy} dx dy = .09333 \\ V(X) &= E(X^2) - [E(X)]^2 = .09333 - (.3)^2 = .00333 \\ \sigma_X &= \sqrt{V(X)} = \sqrt{.00333} = .057735 \\ E(Y^2) &= \int_0^\infty \int_{.2}^{.4} 5xy^2 e^{-xy} dx dy = 25 \\ V(Y) &= E(Y^2) - [E(Y)]^2 = 25 - (3.465736)^2 = 12.98867 \\ \sigma_Y &= \sqrt{V(Y)} = \sqrt{12.98867} = 3.60398 \end{aligned}$$

We now have all the pieces to calculate the correlation, which is given by

$$\text{Corr}(X, Y) = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y} = \frac{-0.03972}{(.057735)(3.60398)} = -0.19089$$

Mathematics 326: Mathematical Statistics

EXAMPLE: Find the Posterior Mean and Variance for θ (in terms of the Y_i 's and α, β) given that $Y_1, \dots, Y_n \sim iid$ Exponential(θ) and θ has a prior distribution of an Inverse-Gamma(α, β), where

$$p(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{-(\alpha+1)} e^{-\frac{\beta}{\theta}}.$$

Note that for the $Y \sim$ Inverse-Gamma(α, β), $E[Y] = \beta(\alpha - 1)^{-1}$ and $V[Y] = \beta^2(\alpha - 1)^{-2}(\alpha - 2)^{-1}$.

Solution: To begin this problem, we must remember that the posterior distribution is given by the following

$$g(\theta|Y_1, \dots, Y_n) = \frac{f(Y_1, \dots, Y_n|\theta) \cdot p(\theta)}{h(Y_1, \dots, Y_n)} = \frac{f(Y_1, \dots, Y_n|\theta) \cdot p(\theta)}{\int f(Y_1, \dots, Y_n|\theta) \cdot p(\theta) d\theta}.$$

Using the information stated in the problem, we can see that

$$\begin{aligned} f(Y_1, \dots, Y_n|\theta) &= \prod_{i=1}^n \frac{1}{\theta} e^{-\frac{y_i}{\theta}} \\ &= \left(\frac{1}{\theta}\right)^n e^{-\frac{\sum Y_i}{\theta}} \end{aligned}$$

and

$$p(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{-(\alpha+1)} e^{-\frac{\beta}{\theta}}.$$

So we now have everything we need to derive the posterior distribution, which is calculated as

$$g(\theta|Y_1, \dots, Y_n) = \frac{\left(\frac{1}{\theta}\right)^n e^{-\frac{\sum Y_i}{\theta}} \left(\frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{-(\alpha+1)} e^{-\frac{\beta}{\theta}}\right)}{\int_0^\infty \left(\left(\frac{1}{\theta}\right)^n e^{-\frac{\sum Y_i}{\theta}} \left(\frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{-(\alpha+1)} e^{-\frac{\beta}{\theta}}\right)\right) d\theta}$$

and this is proportional to $(\propto) f(Y_1, \dots, Y_n|\theta) \cdot p(\theta)$, so we get our final posterior distribution to look like the following equation

$$\begin{aligned} g(\theta|Y_1, \dots, Y_n) &= \left(\frac{1}{\theta}\right)^n e^{-\frac{\sum Y_i}{\theta}} \left(\frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{-(\alpha+1)} e^{-\frac{\beta}{\theta}}\right) \\ &= \frac{\beta^\alpha}{\Gamma(\alpha)} e^{-\frac{\sum Y_i + \beta}{\theta}} \theta^{-\alpha-1-n}. \end{aligned}$$

We can see above that our distribution looks to be an Inverse-Gamma where the parameters are $\alpha' = \alpha + n$ and $\beta' = \sum Y_i + \beta$. From this we can derive the Posterior Mean and Variance using the given formulas for Inverse-Gamma distributions. We find that

$$\text{posterior distribution} \sim \text{Inverse-Gamma} \left(\alpha + n, \sum Y_i + \beta \right)$$

therefore

$$\text{Posterior Mean} = \sum Y_i + \beta ((\alpha + n) - 1)^{-1}$$

and

$$\text{Posterior Variance} = \left(\sum Y_i + \beta \right)^2 ((\alpha + n) - 1)^{-2} ((\alpha + n) - 2)^{-1}.$$

Appendix A

L^AT_EX Symbols

Symbols

Math

<code>\sum</code>	\sum	<code>\prod</code>	\prod	<code>\coprod</code>	\coprod	<code>\int</code>	\int
<code>\oint</code>	\oint	<code>\bigcap</code>	\bigcap	<code>\bigcup</code>	\bigcup	<code>\bigsqcup</code>	\bigsqcup
<code>\bigvee</code>	\bigvee	<code>\bigwedge</code>	\bigwedge	<code>\bigodot</code>	\bigodot	<code>\bigotimes</code>	\bigotimes
		<code>\bigoplus</code>	\bigoplus	<code>\biguplus</code>	\biguplus		
<code>\hat{a}</code>	\hat{a}	<code>\check{a}</code>	\check{a}	<code>\breve{a}</code>	\breve{a}	<code>\acute{a}</code>	\acute{a}
<code>\tilde{a}</code>	\tilde{a}	<code>\bar{a}</code>	\bar{a}	<code>\vec{a}</code>	\vec{a}	<code>\dot{a}</code>	\dot{a}
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<code>\widetilde{abc}</code>	\widetilde{abc}	<code>\widehat{abc}</code>	\widehat{abc}	<code>\overleftarrow{abc}</code>	\overleftarrow{abc}		
<code>\overrightarrow{abc}</code>	\overrightarrow{abc}	<code>\overline{abc}</code>	\overline{abc}	<code>\overbrace{abc}</code>	\overbrace{abc}		
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<code>\sqrt[n]{abc}</code>	$\sqrt[n]{abc}$	<code>f'</code>	f'	<code>\frac{abc}{xyz}</code>	$\frac{abc}{xyz}$		
<code>x^{k}</code>	x^k	<code>\mathbf{B}</code>	\mathbf{B}	<code>\mathbb{R}</code>	\mathbb{R}		
<code>x_{k}</code>	x_k	<code>\mathcal{C}</code>	\mathcal{C}	<code>\mathrm{T}</code>	T		

Greek

<code>\Gamma</code>	Γ	<code>\Delta</code>	Δ	<code>\Theta</code>	Θ	<code>\Lambda</code>	Λ
<code>\Xi</code>	Ξ	<code>\Pi</code>	Π	<code>\Sigma</code>	Σ	<code>\Upsilon</code>	Υ
<code>\Phi</code>	Φ	<code>\Psi</code>	Ψ	<code>\Omega</code>	Ω	<code>\\$</code>	$\$$

<code>\alpha</code>	α	<code>\beta</code>	β	<code>\gamma</code>	γ	<code>\delta</code>	δ
<code>\epsilon</code>	ϵ	<code>\varepsilon</code>	ε	<code>\zeta</code>	ζ	<code>\eta</code>	η
<code>\theta</code>	θ	<code>\vartheta</code>	ϑ	<code>\iota</code>	ι	<code>\kappa</code>	κ
<code>\lambda</code>	λ	<code>\mu</code>	μ	<code>\nu</code>	ν	<code>\xi</code>	ξ
<code>o</code>	o	<code>\pi</code>	π	<code>\varpi</code>	ϖ	<code>\rho</code>	ρ
<code>\varrho</code>	ϱ	<code>\sigma</code>	σ	<code>\varsigma</code>	ς	<code>\tau</code>	τ
<code>v</code>	v	<code>\phi</code>	ϕ	<code>\varphi</code>	φ	<code>\chi</code>	χ
		<code>\psi</code>	ψ	<code>\omega</code>	ω		
		<code>\digamma</code>	F	<code>\beth</code>	\beth		
		<code>\gimel</code>	\beth	<code>\daleth</code>	\beth		

Symbols

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<code>\ell</code>	ℓ	<code>\wp</code>	\wp	<code>\Re</code>	\Re	<code>\Im</code>	\Im
<code>\prime</code>	\prime	<code>\emptyset</code>	\emptyset	<code>\angle</code>	\angle	<code>\infty</code>	∞
<code>\partial</code>	∂	<code>\nabla</code>	∇	<code>\triangle</code>	\triangle	<code>\forall</code>	\forall
<code>\exists</code>	\exists	<code>\neg</code>	\neg	<code>\surd</code>	\surd	<code>\top</code>	\top
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International

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Typeface

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Functions(x)...

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\in	\in	\ni	\ni	\vdash	\dashv	\dashv

\equiv	\equiv	\sim	\sim	\simeq	\simeq	\asymp	\asymp
\approx	\approx	\cong	\cong	\neq	\neq	\doteq	\doteq
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AMS

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<code>\sqsupset</code>	\sqsupset	<code>\preccurlyeq</code>	\preccurlyeq	<code>\succcurlyeq</code>	\succcurlyeq
<code>\curlyeqprec</code>	\curlyeqprec	<code>\curlyeqsucc</code>	\curlyeqsucc	<code>\precsim</code>	\precsim
<code>\succsim</code>	\succsim	<code>\precapprox</code>	\precapprox	<code>\smallfrown</code>	\frown
<code>\bumpeq</code>	\bumpeq	<code>\Bumpeq</code>	\Bumpeq	<code>\shortmid</code>	\shortmid
<code>\shortparallel</code>	\parallel	<code>\between</code>	\between	<code>\pitchfork</code>	\pitchfork
<code>\varpropto</code>	\propto	<code>\backepsilon</code>	\backepsilon		

AMS NOT = < >

<code>\nless</code>	\nless	<code>\ngtr</code>	\ngtr	<code>\nleq</code>	\nleq
<code>\ngeq</code>	\ngeq	<code>\nleqslant</code>	\nleqslant	<code>\ngeqslant</code>	\ngeqslant
<code>\nleqq</code>	\nleqq	<code>\ngeqq</code>	\ngeqq	<code>\lneq</code>	\lneq
<code>\gneq</code>	\gneq	<code>\npreceq</code>	\npreceq	<code>\nsucceq</code>	\nsucceq
<code>\precneqq</code>	\precneqq	<code>\succneqq</code>	\succneqq	<code>\precnsim</code>	\precnsim
<code>\succnsim</code>	\succnsim	<code>\precnapprox</code>	\precnapprox	<code>\succnapprox</code>	\succnapprox
<code>\nsim</code>	\nsim	<code>\ncong</code>	\ncong	<code>\ntrianglelefteq</code>	\ntrianglelefteq
<code>\ntrianglerighteq</code>	\ntrianglerighteq	<code>\subsetseteq</code>	\subsetseteq	<code>\nsupseteq</code>	\nsupseteq
<code>\subsetseteqq</code>	\subsetseteqq	<code>\nsupseteqq</code>	\nsupseteqq	<code>\subsetsetneq</code>	\subsetsetneq
<code>\lneqq</code>	\lneqq	<code>\gneqq</code>	\gneqq	<code>\lvertneqq</code>	\lvertneqq
<code>\gvertneqq</code>	\gvertneqq	<code>\lnsim</code>	\lnsim	<code>\gnsim</code>	\gnsim
<code>\lnapprox</code>	\lnapprox	<code>\gnapprox</code>	\gnapprox	<code>\nprec</code>	\nprec
<code>\nsucc</code>	\nsucc	<code>\nshortmid</code>	\nshortmid	<code>\nshortparallel</code>	\nshortparallel
<code>\nmid</code>	\nmid	<code>\nparallel</code>	\nparallel	<code>\nvdash</code>	\nvdash
<code>\nvDash</code>	\nvDash	<code>\nVdash</code>	\nVdash	<code>\nVDash</code>	\nVDash
<code>\ntriangleleft</code>	\ntriangleleft	<code>\ntriangleright</code>	\ntriangleright	<code>\supsetsetneq</code>	\supsetsetneq
<code>\varsubsetneq</code>	\varsubsetneq	<code>\varsupsetneq</code>	\varsupsetneq	<code>\subsetsetneqq</code>	\subsetsetneqq
<code>\supsetsetneqq</code>	\supsetsetneqq	<code>\varsubsetneqq</code>	\varsubsetneqq	<code>\varsupsetneqq</code>	\varsupsetneqq

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